Dense Matter EoS and applications in Core Collapse SuperNovae and Neutron Stars

Francesca Gulminelli - LPC Caen, France

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« Dense matter »: as dense as nuclei
« Equation of State »: $p(\rho,T)$
Here: $p(\rho_n, \rho_p,T)$ or also $\varepsilon(\rho_n, \rho_p,T)$

Lecture I: the Equation of State of compact stars

1. Dense matter in the universe and theoretical challenges
2. Modelling the EoS in the mean-field approximation
   a. density functional approaches
   b. effective lagrangians
   c. pairing correlations
3. Constraining the parameters
4. Phase transitions in dense matter
   a. from core to crust
   b. from nucleons to quarks

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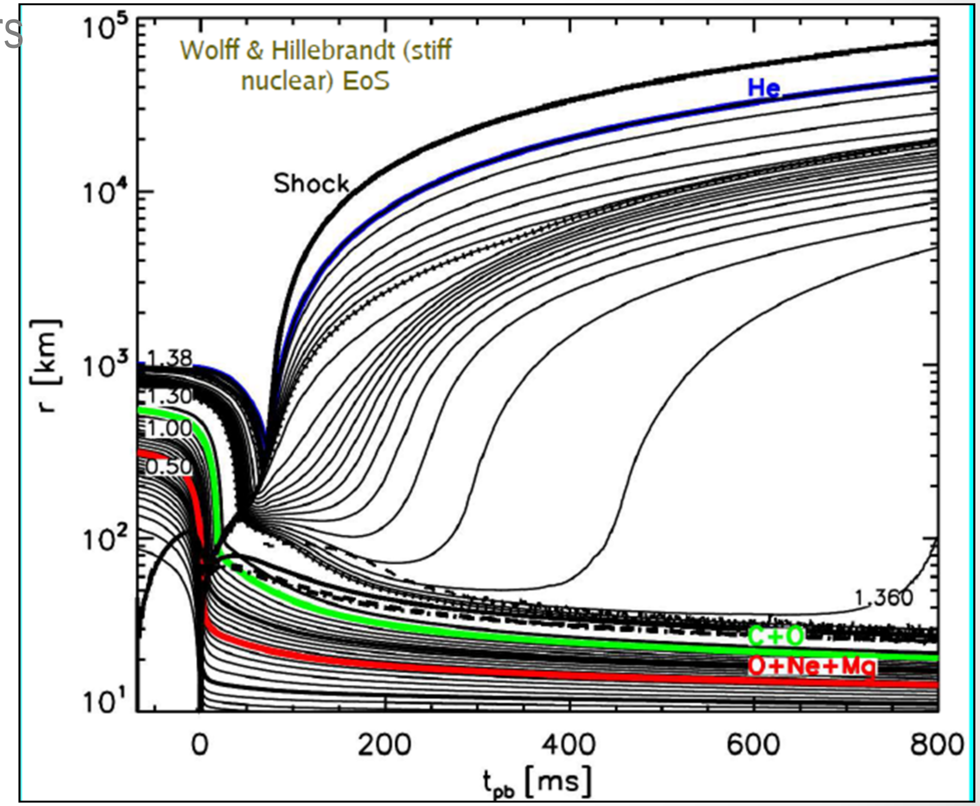
I. Dense matter in the universe

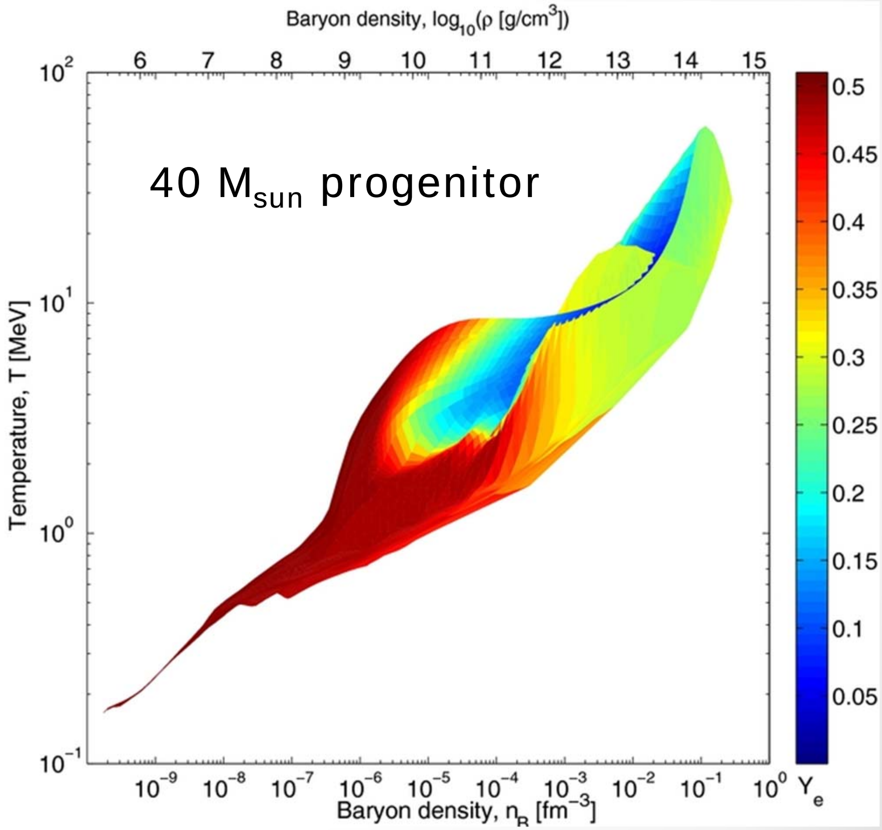
- Supernova explosion occurs via core-collapse in very massive stars ($M > 8M_{sun}$)



F.S.Kitaura et al, A&A 450 (06) 345

I. Dense matter in the universe

- Supernova explosion occurs via core-collapse in very massive stars ($M>8M_{sun}$)
- $10^6 < \rho < 10^{15}$ g/cm^3
  $0.01<T<50$ MeV in the core

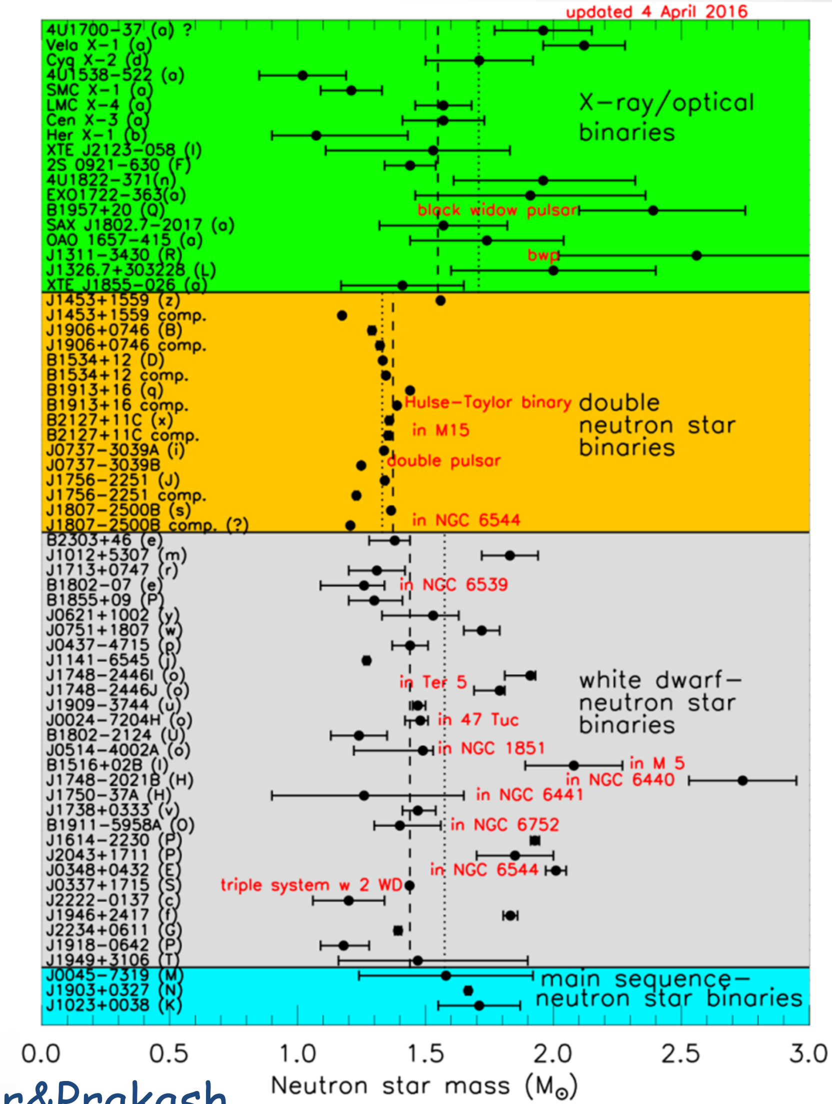


T.Fischer et al, 2011 ApJS **194** 39

I. Dense matter in the universe

- Supernova explosion occurs via core-collapse in very massive stars ($M>8M_{sun}$)
- $10^6 < \rho < 10^{15}$ g/cm^3
  0.01<T<50 MeV in the core
- The density in the residual pulsar (neutron star) is of the same order $<\rho> \sim \rho_0 \sim 10^{14}$ g/cm^3

=> Matter with nucleonic or sub-nucleonic dof !

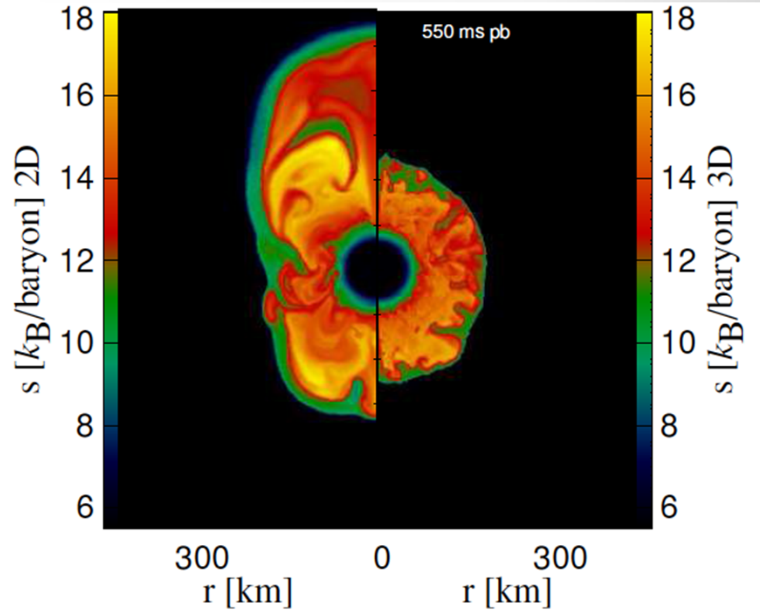


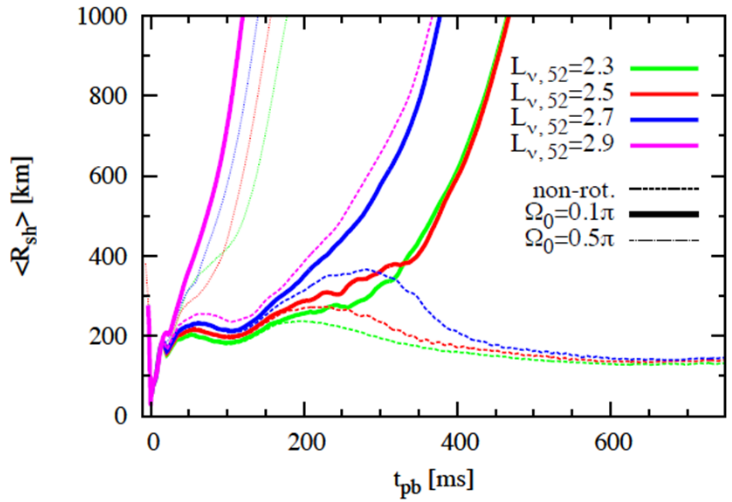
Compilation by Lattimer&Prakash

BIG challenges for theory

1. Present best 3D hydro simulations do not yet produce satisfactory explosions
   - Incertainty in the initial conditions

Hanke et al 2012

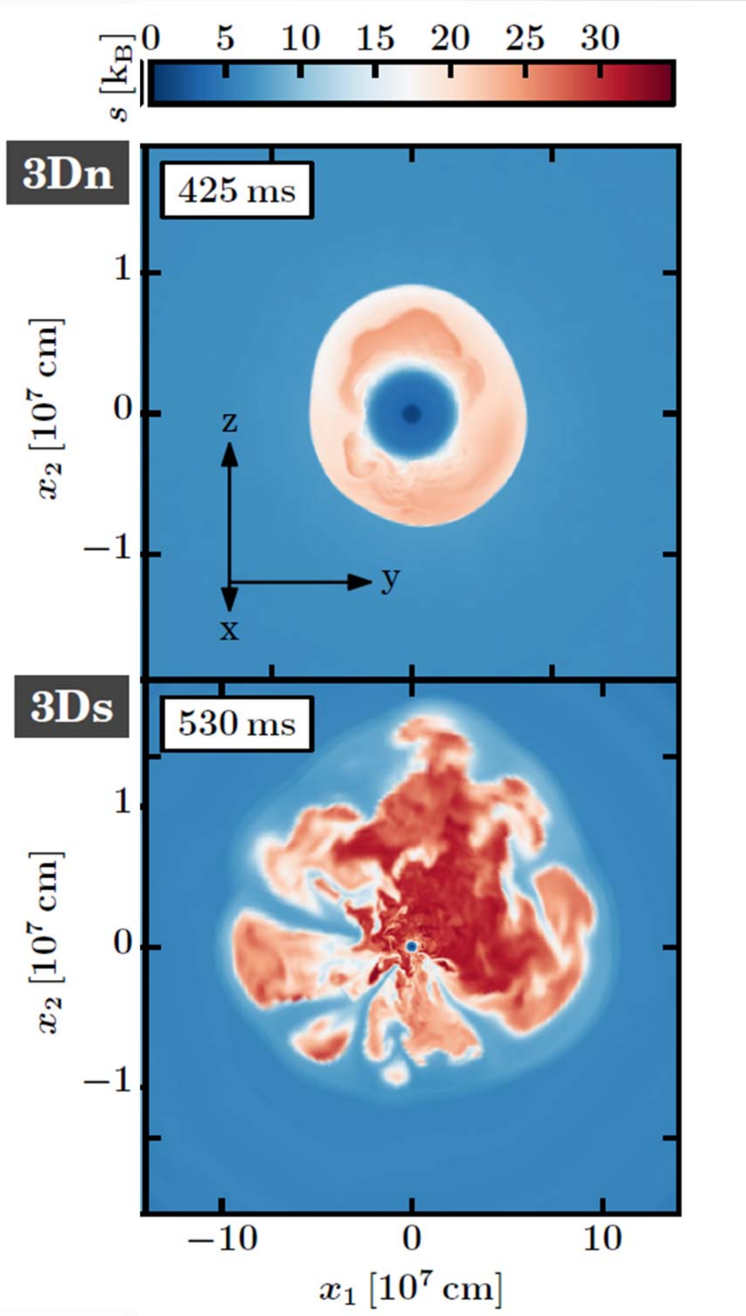




Nakamura et al 2014

BIG challenges for theory

1. Present best 3D hydro simulations do not yet produce satisfactory explosions
   - Incertainty in the initial conditions
   - Incertainty in the ν dynamics



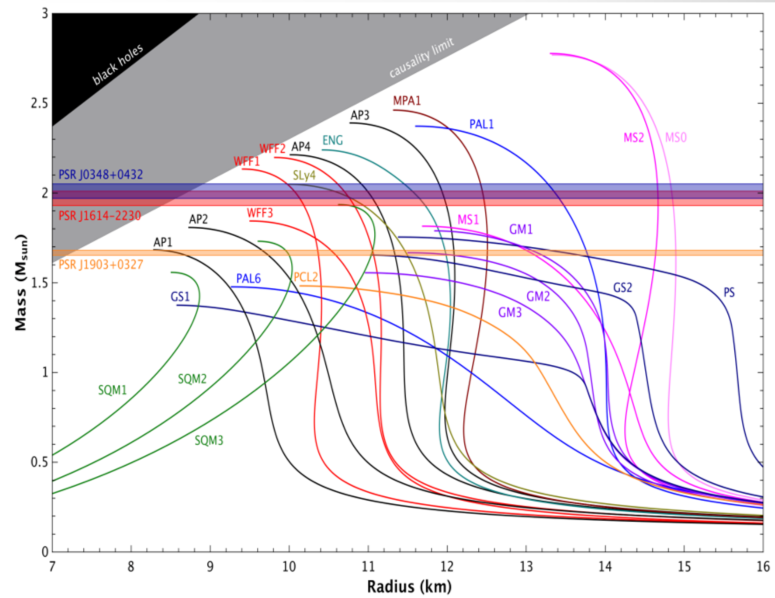
Melson et al. 2015

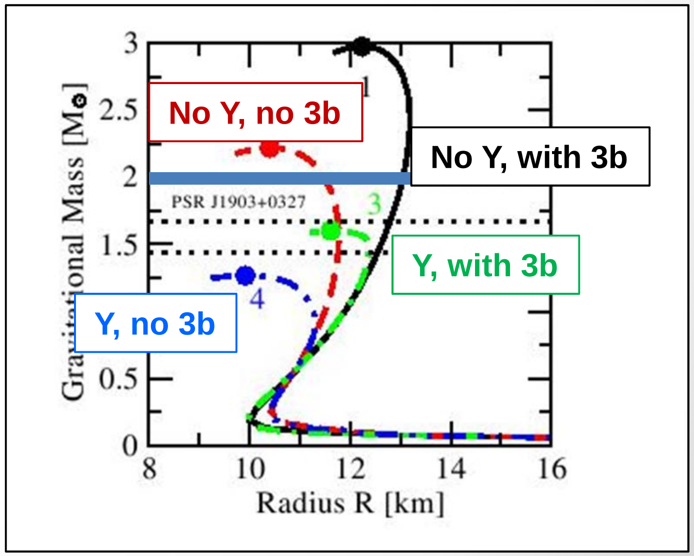
- **=> Nuclear physics essential !**

BIG challenges for theory

P. Demorest et al., Nature 467 1081 (2010).
J. Antoniadis et al., Science, 340, 6131 (2013).

2. Present best EoS modelling cannot yet explain the most massive NS



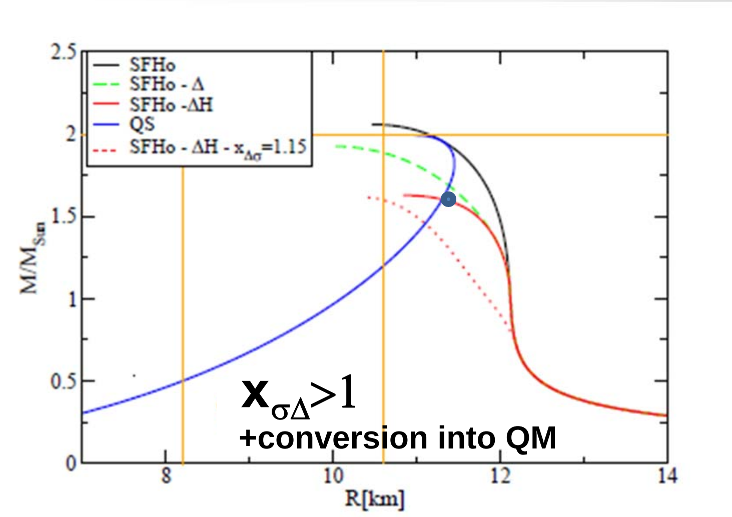


I.Vidana et al, Europhys.Lett.94:11002,2011

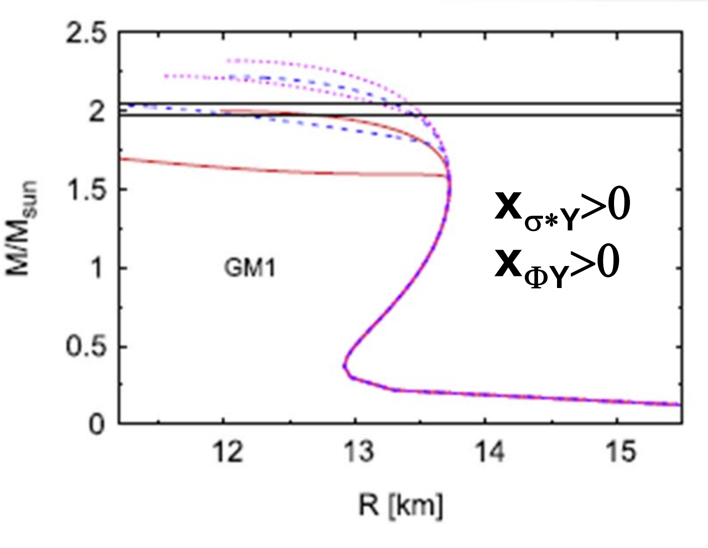
BIG challenges for theory

2. Present best EoS modelling cannot yet explain the most massive NS
   - Strangeness couplings at high density ?
   - Transition to quark matter ?

A.Drago et al 2014



M.Oertel et al 2015



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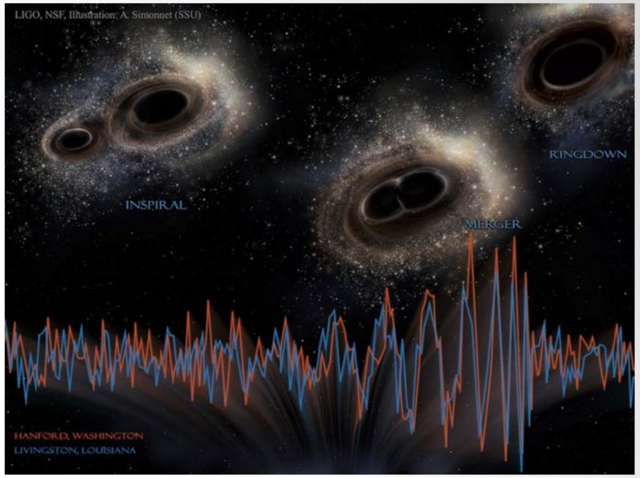
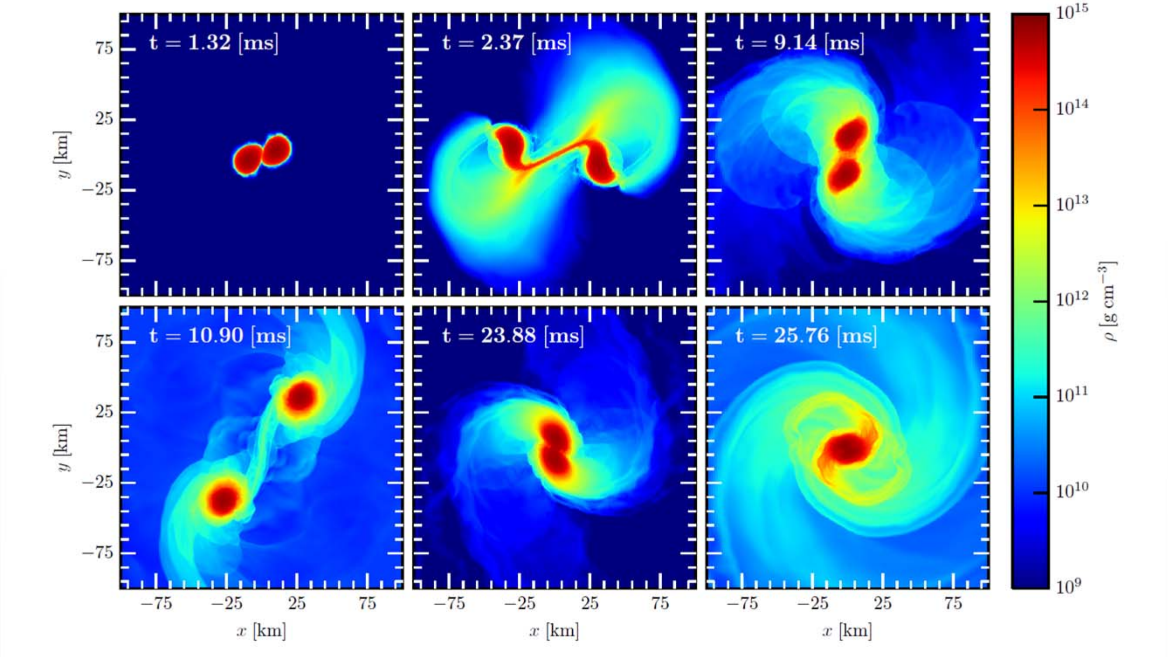


3. The recent detection of GW by aLIGO opens an exciting avenue of GW observation from NS
   - Continuous GW from deformed NS
   - R-modes in young sources
   - Binary NS merging

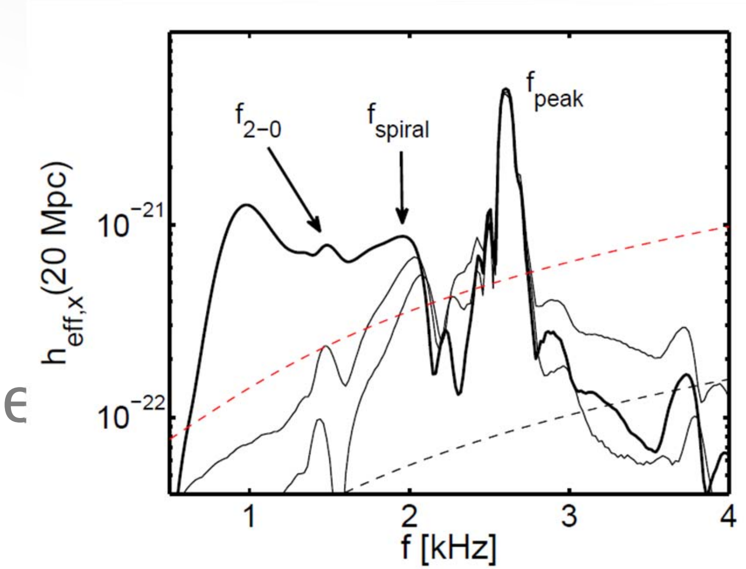
A.Radice et al ArXiV 1601.02426



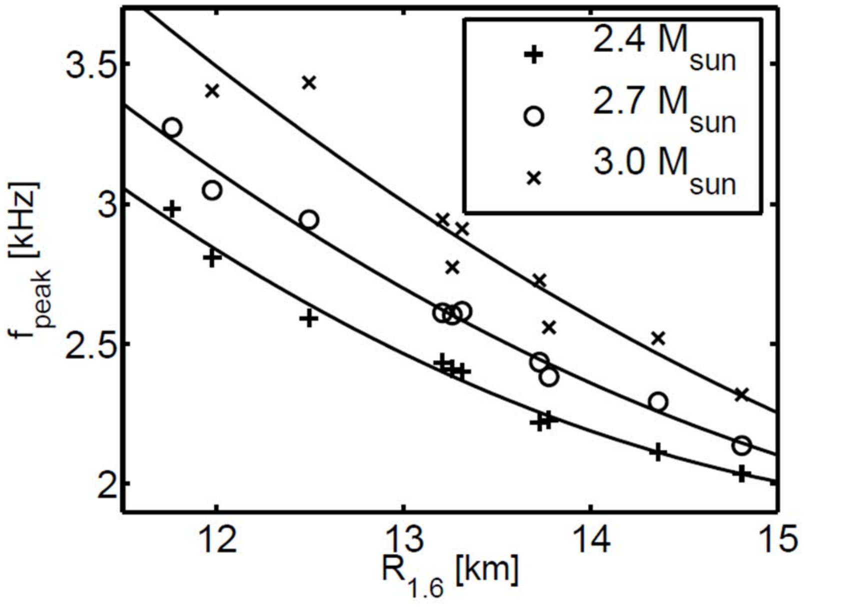
BIG challenges for theory

3. The recent detection of GW by aLIGO opens an exciting avenue of GW observation from NS
   - Continuous GW from deformed NS
   - R-modes in young sources
   - **Binary NS merging**

   Detectable (40 events/year) oscillations (f-mode) of the post-merger remnant are correlated to the EoS (here expressed as $R_{1.6}$)



A.Bauswein, arXiV:1508.05493



- **=> Nuclear physics essential !**

Lecture I: the Equation of State of compact stars

1. Dense matter in the universe and theoretical challenges
2. **Modelling the EoS in the mean-field approximation**
   a. **density functional approaches**
   b. **effective lagrangians**
   c. **pairing correlations**
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   a. from core to crust
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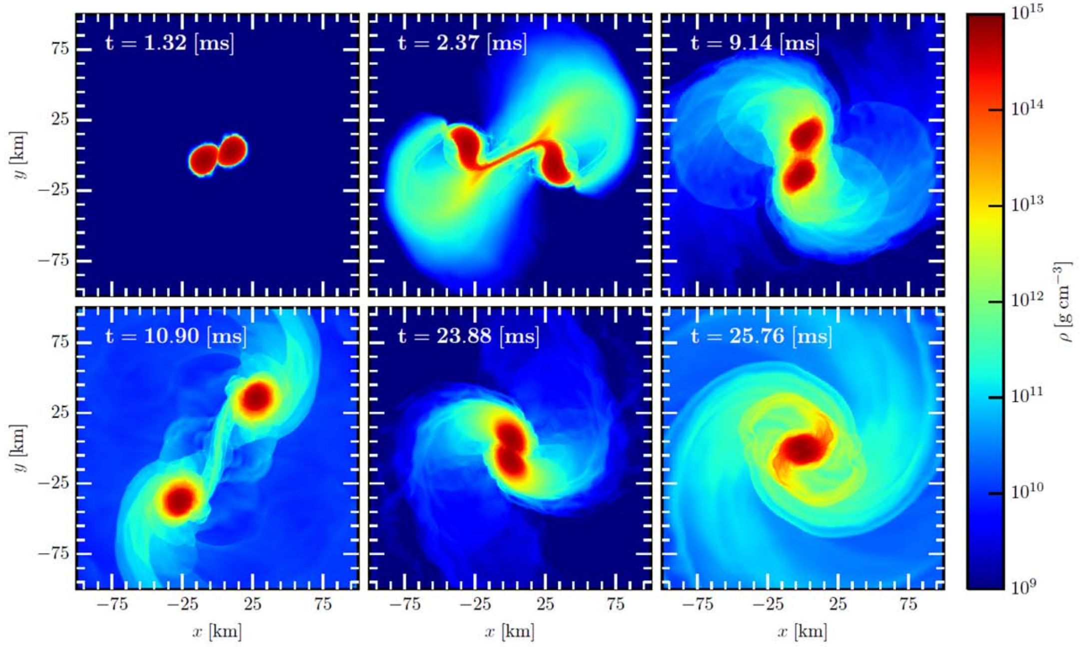
2. Modelling the EoS in the mean field approximation

- **Thermodynamic limit** $\forall \boldsymbol{r}$ ($\sim 10^{38}$ particles/cm^3)

$\Rightarrow$ Homogeneous $\rho_q(r) = \rho_q$ ($\forall$ q constituent)



2. Modelling the EoS in the mean field approximation

- **Thermodynamic limit** $\forall \boldsymbol{r}$ ($\sim 10^{38}$ particles/cm^3)

$\Rightarrow$ Homogeneous $\rho_q(r) = \rho_q$ ($\forall$ q constituent)

$\Rightarrow \varepsilon_{tot} = \varepsilon_B + \varepsilon_L$ (baryons and leptons decoupled)

$\Rightarrow$ Translational invariance: $V_q(r)=cst$

$\Rightarrow (\hat{t}_q + \hat{V}_q)|i\rangle = e_i|i\rangle \quad \langle r|i\rangle = \frac{1}{(2\pi)^3} e^{ik_i \cdot r}$ plane waves

$\Rightarrow e_q(k) = \sqrt{m_q^2 + k^2} + V_q(\rho_q, \rho_{q'})$ single particle energy

$\Rightarrow \varepsilon_q = \varepsilon_{FG} + \int_0^\rho d\rho V_q$ energy density

- **Nucleons only:** $\varepsilon_B = \varepsilon_{FG,p} + \varepsilon_{FG,n} + \boldsymbol{\varepsilon}(\boldsymbol{\rho_n}, \boldsymbol{\rho_p})$ energy functional: the quantity to be calculated.

a- Non relativistic mean-field

The Skyrme approach (zero range effective interaction)

$$\varepsilon_{Skyrme} = \sum_{q=n,p} \frac{1}{\pi^2} \int_0^{p_F^q} dp p^2 \frac{p^2}{2m_q^*} + (C_0\rho^2 + C_3\rho^\gamma) + (D_0\rho^2 + D_3\rho^\gamma)\delta^2$$

$$p_F^q = \hbar\left(3\pi^2\rho_q\right)^{1/3}$$

$$\begin{cases} \rho = \rho_n + \rho_p \\ \delta = \dfrac{\rho_n - \rho_p}{\rho} \end{cases}$$

The finite range approach (Gogny, M3Y..)

$$\varepsilon_{M3Y} = \sum_{q=n,p} \frac{1}{\pi^2} \int_0^{p_F^q} dp p^2 \frac{p^2}{2m_q^*} + (J_{v00} + J_{v01}\delta^2)(\rho^2 - \beta\rho^\gamma)$$

Same functional dependence as Skyrme!
The interaction range plays no role for nuclear matter

Proof: $$\varepsilon = \frac{1}{2V} \sum_{ij} \langle ij|\hat{v}|ij - ji\rangle = \langle v\rangle \rho^2$$

$$< \vec{r}|i > \propto \frac{1}{\sqrt{V}} e^{i\vec{k}.\vec{r}}$$

$$\langle v\rangle = \frac{1}{2A^2} \sum_{i=1}^{A} \sum_{j=1}^{A} \int d^3s\, v(s)\left(1 - e^{-i\Delta\vec{k}\cdot\vec{s}}\right) = t_0$$

If $v(s) = t_0\delta(s)$

a- Non relativistic mean-field

- In principle, functional form well established and parameters constrained by nuclear experiments

- BUT ad-hoc density dependent terms $\propto \rho^{\gamma}$ which simulate many body effects

$\Rightarrow$ Arbitrariness in the functional form

$\Rightarrow$ Arbitrariness in the extrapolations!

b- Relativistic mean-field

$$\varepsilon_{RMF} = \sum_{q=n,p} \frac{1}{\pi^2} \int_0^{p_{Fq}} dp p^2 \sqrt{p^2 + m_q^{*2}}$$

Kinetic energy

$$\begin{cases} m^* = m - g_s \sigma_0 \; ; \sigma_0 = \dfrac{g_s}{m_\sigma^2} \rho_s \\ \rho_s = \sum_{q=n,p} \dfrac{1}{\pi^2} \int_0^{p_{Fq}} dp p^2 \dfrac{m_q^{*2}}{\sqrt{p^2 + m_q^{*2}}} \end{cases}$$

$$+ g_v \omega_0 \rho + \frac{1}{2} g_\rho b_0 \rho \delta$$

Baryon-meson coupling $\propto \rho^2$

$$+ \frac{1}{2}\left(m_\sigma^2 \sigma_0^2 - m_\omega^2 \omega_0^2 - m_\rho^2 b_0^2 \right)$$

Meson fields $\propto \rho^2$

$$\omega_0 = \frac{g_v}{m_\omega^2} \rho \quad ; \quad b_0 = \frac{g_\rho}{m_\rho^2} \rho \delta$$

- In principle, functional form controlled by the underlying effective Lagrangian

$$\mathcal{L}_q = \bar{\psi}_q \left[\gamma_\mu \left(i\partial^\mu - g_v \omega^\mu - \frac{1}{2} g_\rho \vec{\tau} \cdot \vec{b}^\mu \right) - m - g_s \sigma \right] \psi_q$$

b- Relativistic mean-field

$$\mathcal{L}_q = \bar{\psi}_q \left[\gamma_\mu \left(i\partial^\mu - g_v \omega^\mu - \frac{1}{2} g_\rho \vec{\tau}\cdot\vec{b}^\mu\right) - m - g_s\sigma\right]\psi_q$$

- However, the mapping is broken by ad-hoc density dependent couplings g(ρ) (or non-linear couplings) which simulate many body effects

⇒ Arbitrariness in the functional form

⇒ Arbitrariness in the extrapolations

Relativistic or not ???
It is just a question of taste......

The biggest issue is the determination of the parameters of ε in a model independent way

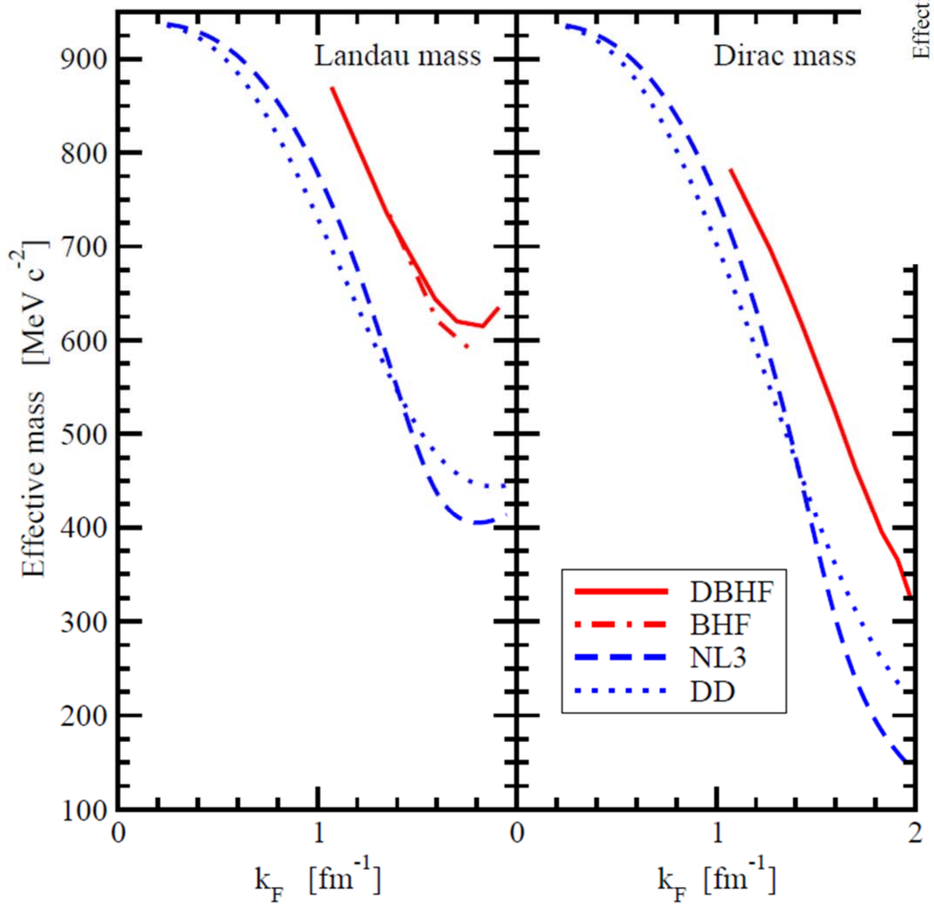
The effective mass issue

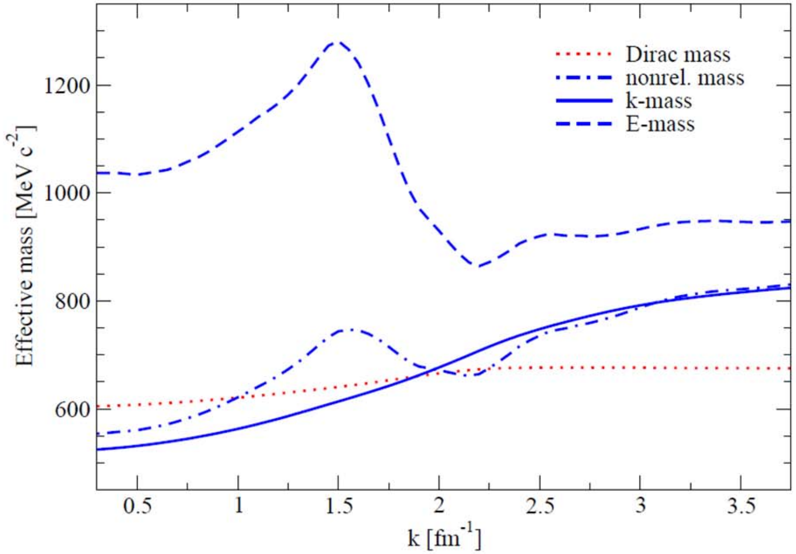
- Nucleons interact with the surrounding medium: their energy=>mass is modified with respect to the vacuum value.
- However, the effective mass m* entering the kinetic energy is not the same in rel and non-rel approaches.

Dirac m*
$$m^* = m + \Re\Sigma(p,\rho) = m - g_s\sigma_0 = m - \frac{g_s^2}{m_\sigma^2}\sum_{q=n,p}\frac{1}{\pi^2}\int_0^{p_{Fq}} dp\, p^2 \frac{m_q^{*2}}{\sqrt{p^2 + m_q^{*2}}}$$

Landau m*
$$m^* = p\left(\frac{de}{dp}\right)^{-1} \qquad e = \frac{p^2}{2m} + \Re U(p,\rho) \qquad \delta\varepsilon = Tr\left(\left(\hat{t} + \hat{U}\right)\delta\hat{\rho}\right)$$

- This leads to a systematic difference in the functional dependence (both isoscalar and isovector)





E.Van Dalen, H.Muther
arXiV:10040144

c - Beyond mean field: pairing correlations

- The attractive part of the residual interaction leads to pairing correlations
- Channels relevant for neutron star matter: $^1S_0(nn, \rho<\rho_0)$, $^3P_2(nn\&pp, \rho>\rho_0)$
- BCS theory:

$$\varepsilon_{tot}(\rho,\delta) = \varepsilon\left(\rho,\delta\right) + \frac{1}{4}\sum_{q=n,p} v_\pi(\rho_q)\tilde{\rho}_q^*\tilde{\rho}_q \qquad \tilde{\rho}_q = 2\frac{\Delta(\rho_q)}{v_\pi(\rho_q)}$$

$$1 = -\frac{v_\pi}{2}\frac{1}{\hbar^3\pi^2}\int_0^{p_{Fq}} dpp^2 \frac{1}{\sqrt{\left(\frac{p^2-p_{Fq}^2}{2m^*}\right)^2+\Delta_q^2}}$$

- Effective interaction optimized to reproduce ab-initio calculations of Δ including polarization and screening effects
- Superfluidity and superconductivity negligible for static properties, but essential for cooling and glitches $C_V \propto exp - \Delta/T$

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3. Constraining the model parameters

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

$$x = \frac{\rho - \rho_0}{\rho_0}$$

$$e = \frac{\varepsilon_B}{\rho}$$

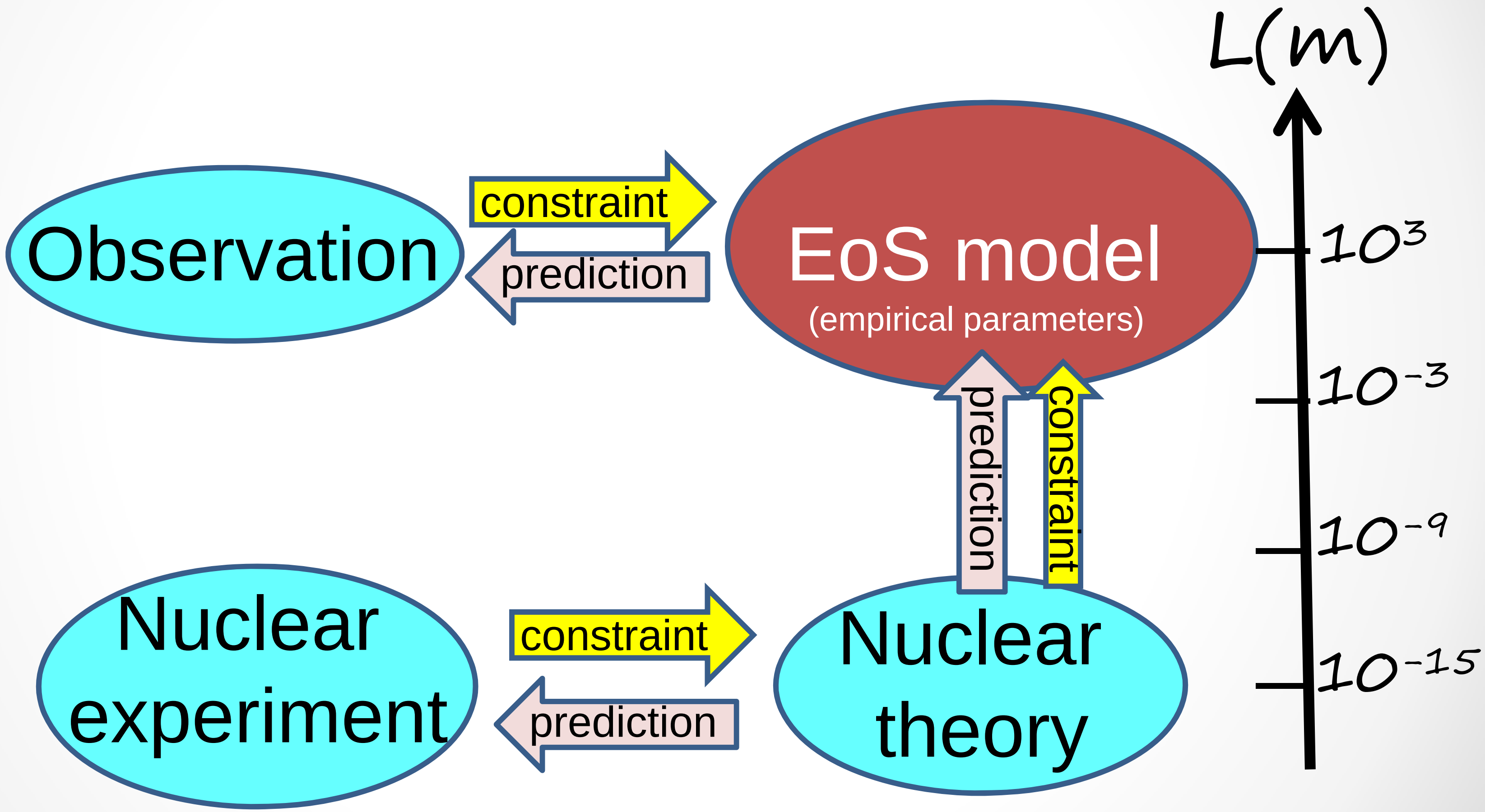
- Definition of empirical parameters
  - Any EoS can be Taylor expanded

$$e(\rho,\delta) = e_{IS}(\rho) + e_{IV}(\rho)\delta^2 + O(\delta^4)$$
$$= \left(\boldsymbol{E_0} + \tfrac{1}{18}\boldsymbol{K_0}x^2 + O(x^3)\right) + \left(\boldsymbol{J_0} + \tfrac{1}{3}\boldsymbol{L}x + \tfrac{1}{18}\boldsymbol{K_{sym}}x^2 + O(x^3)\right)\delta^2$$

  - The actual model parameters can be expressed as a function of the empirical parameters $(E_0, \rho_0, K_0, J_0, L, K_{sym} \dots)$
  - If the empirical parameters are known, the EoS is known
  - If these coefficients are constrained through model comparison with data, **any model** compatible with the constraints can be used to compute compact star properties
  - Data from lab.experiment, observation or ab-initio modelling.

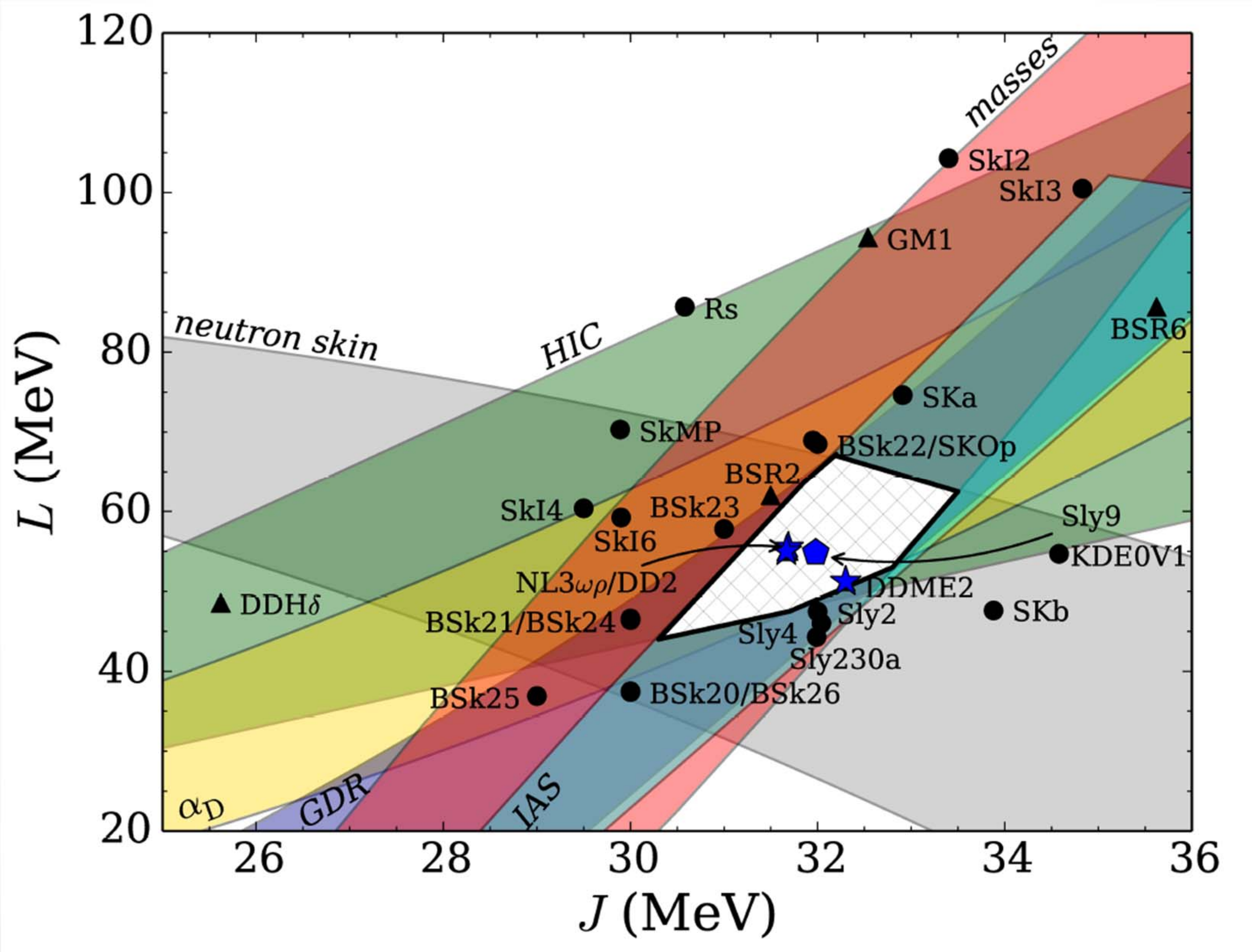
Constraining the empirical parameters:

jumping across the scales!



Constraining the model parameters

a: laboratory experiments

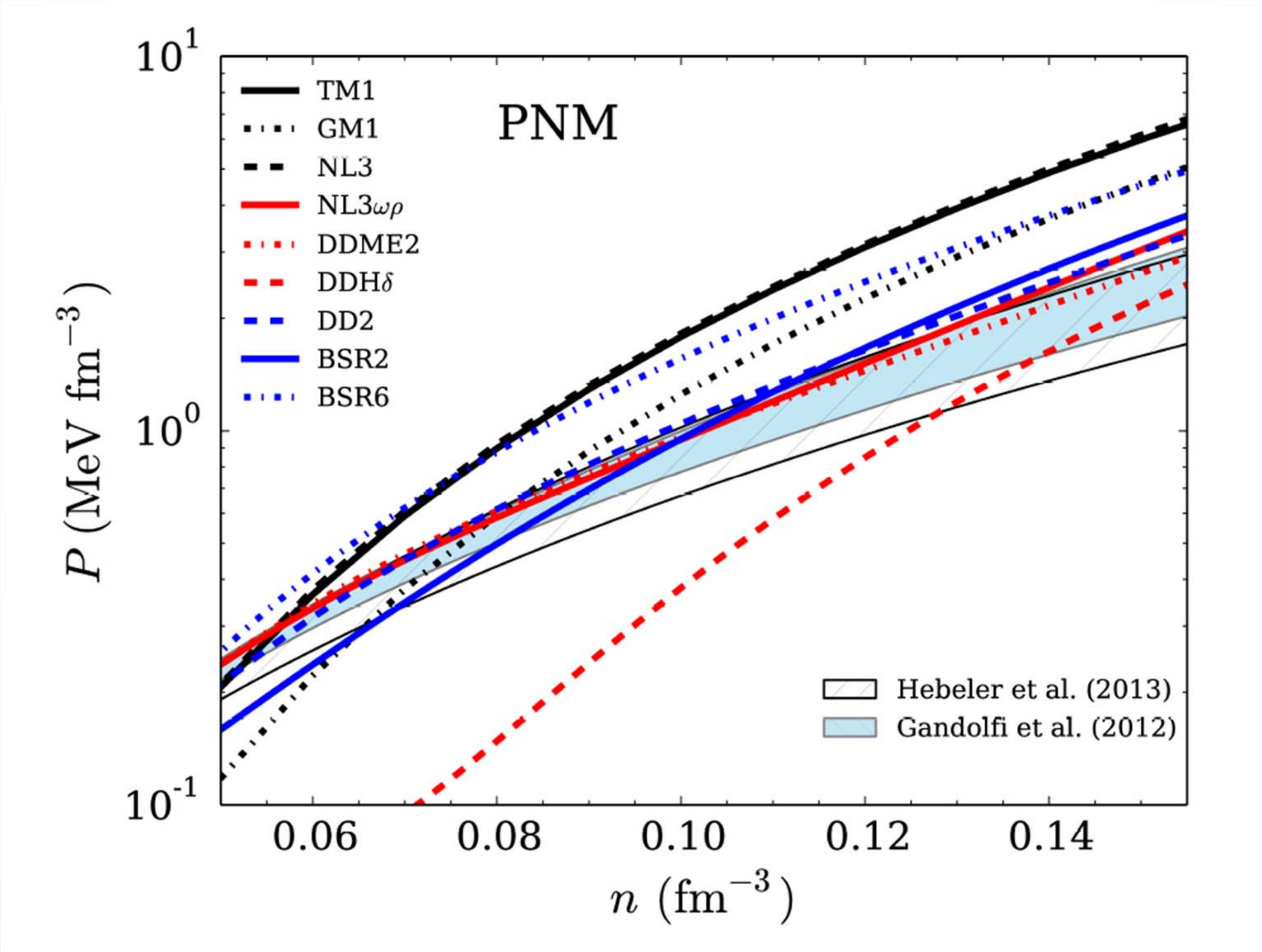


Constraining the model parameters

b: ab-initio modelling

$$P = -\frac{de}{d\rho^{-1}} = \rho\mu - \varepsilon$$

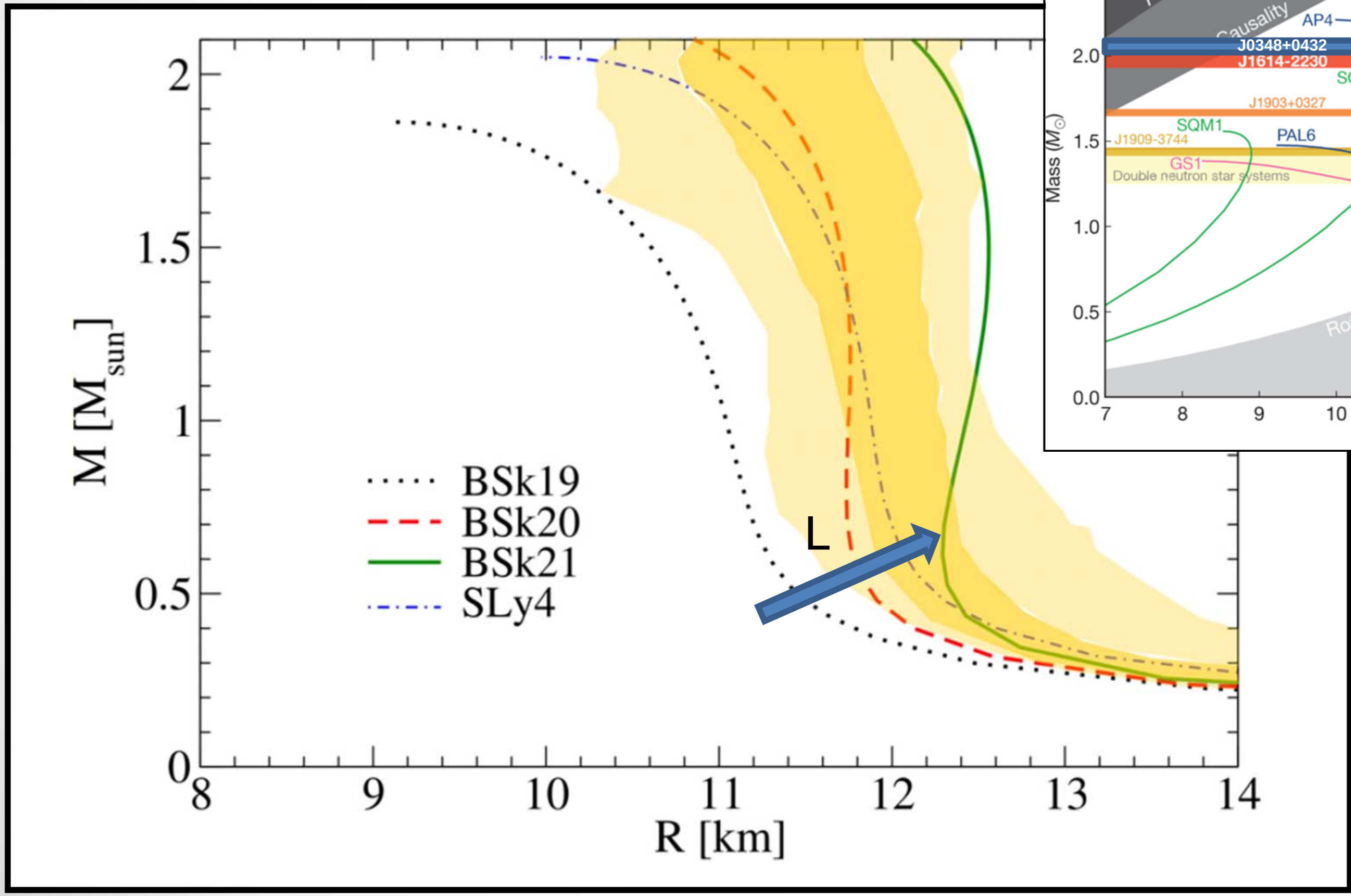
$$P(\rho_0) = \frac{1}{3}\rho_0^2 L - E_0 - J_0$$

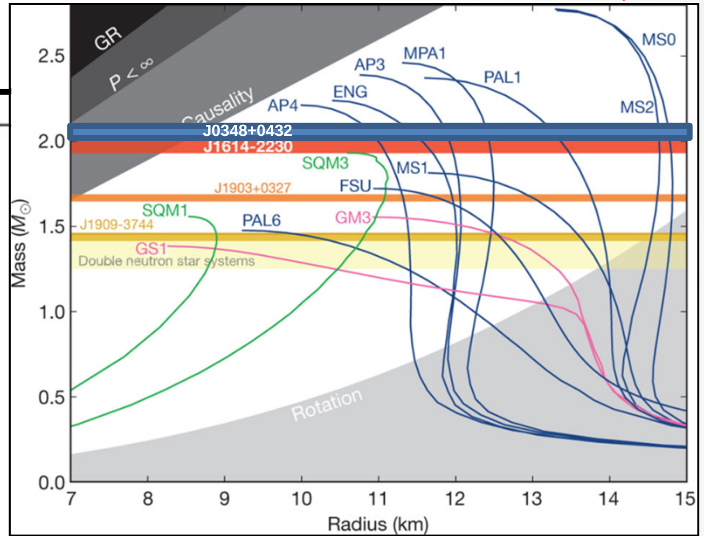


Constraining the model parameters

c: observation

P. Demorest et al., Nature 467 1081 (2010).
J. Antoniadis et al., Science, 340, 6131 (2013).





- A.Fantina et al, A&A 2013

Empirical parameters from various effective approaches

Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
Skyrme	Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23
RMF	Average	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34
RHF	Average	0.1540	-15.97	248.06	389.17	5269.07	33.97	90.03	128.16	523.29	-9955.49
	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74
Average		0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77
σ		0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

$$e_{IS}(\rho) = E_0 + \frac{K_0}{2}x(\rho)^2 + \frac{Q_0}{6}x(\rho)^3 + \ldots,$$

$$e_{IV}(\rho) = E_{sym} + L_{sym}x(\rho) + \frac{K_{sym}}{2}x(\rho)^2 + \frac{Q_{sym}}{6}x(\rho)^3 + \ldots,$$

$$\begin{cases} \rho = \rho_n + \rho_p \\ \delta = (\rho_n - \rho_p)/\rho \end{cases}$$

J.Margueron et al., 2016

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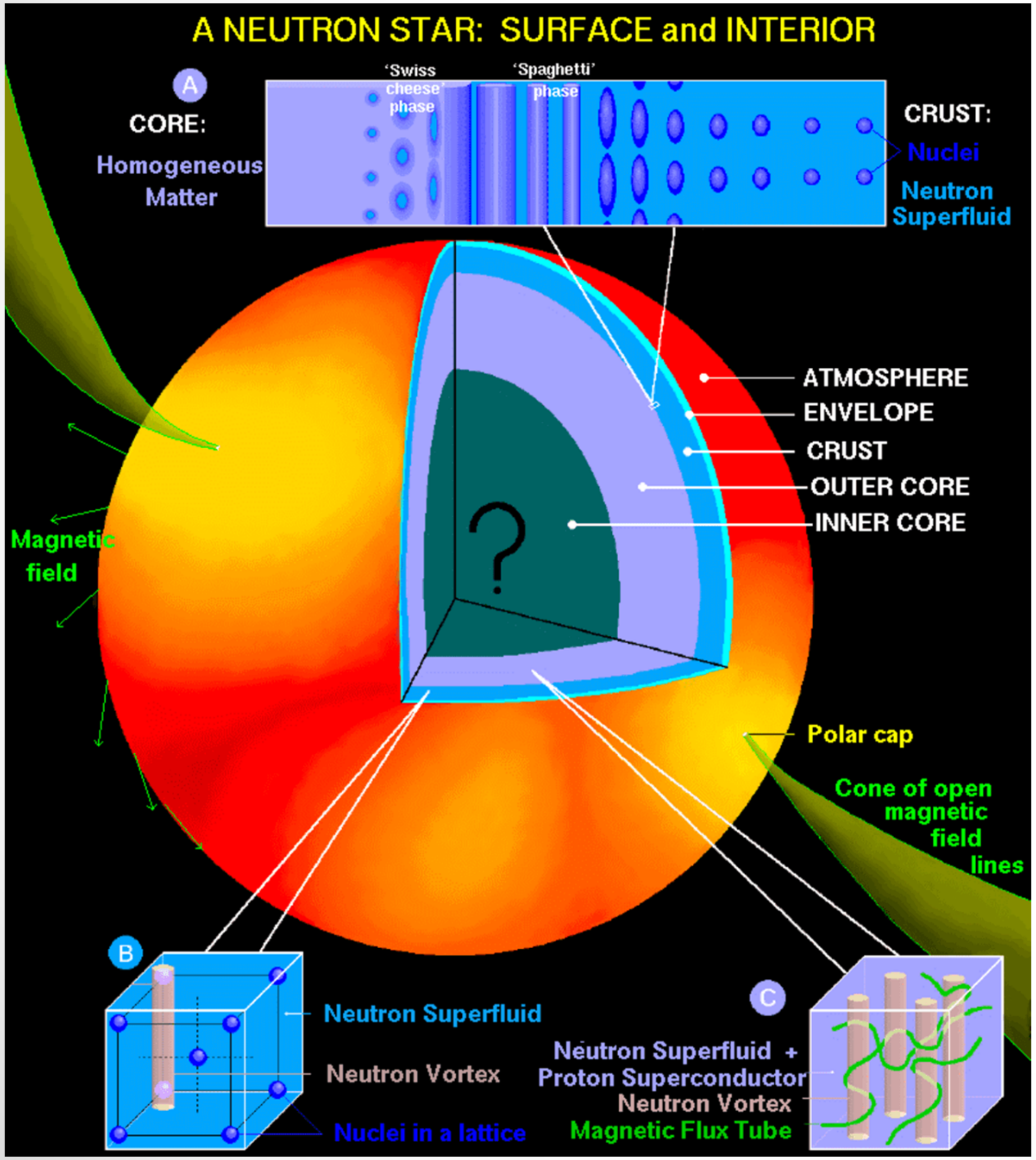
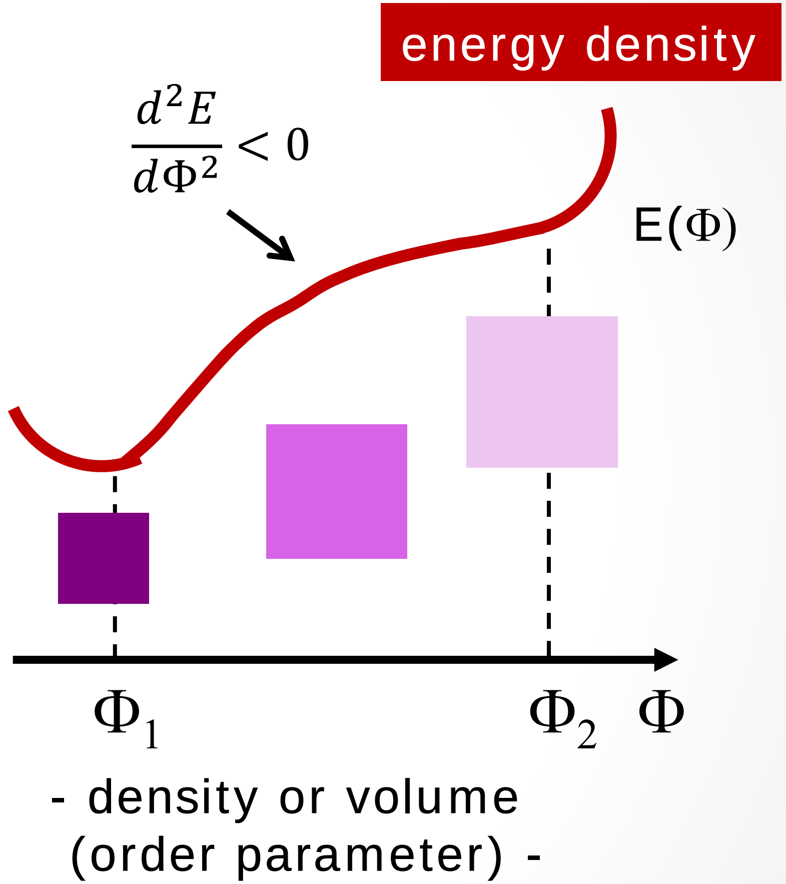
4. Phase transitions in dense matter



Picture: D.Page

Phase transitions: necessarily beyond mean-field

- A mean-field model in the thermodynamic limit implies homogeneous matter $[\hat{h}_i, \vec{k}] = 0$
- Necessarily fails if matter is non-homogeneous
- In MF phase transitions are signalled by instability of homogeneous matter towards phase separation
- => Convexity of the energy functional



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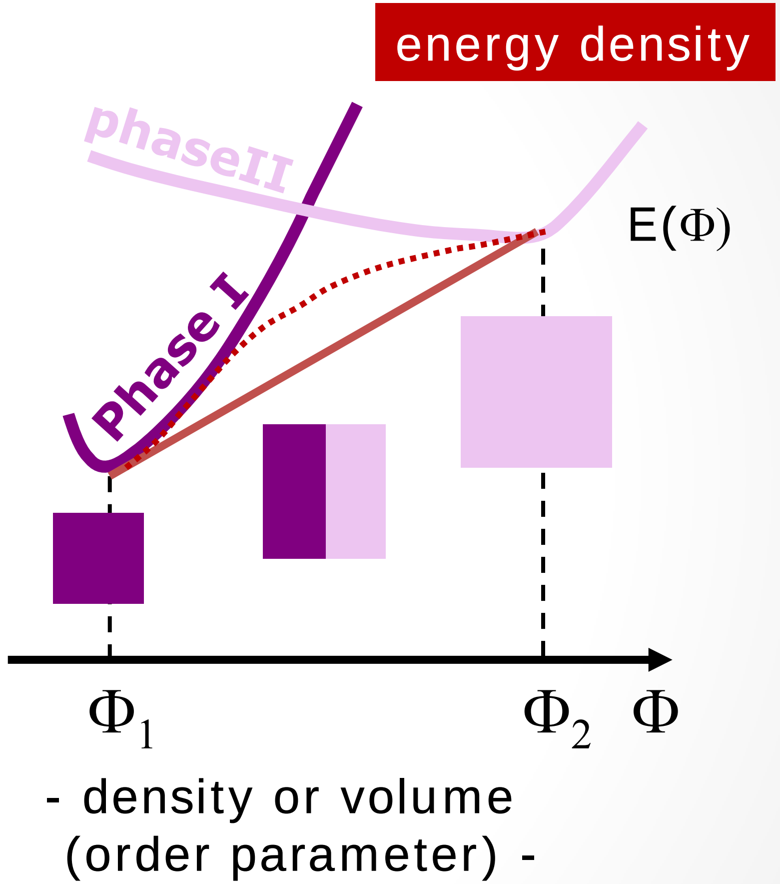
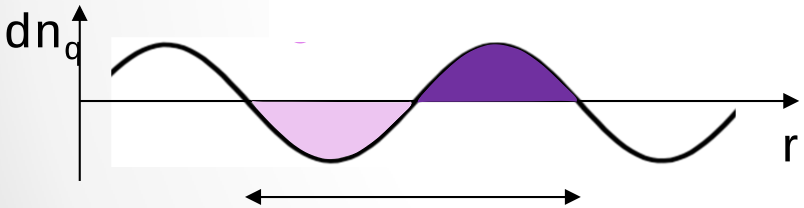


Here: single order parameter (1D space)

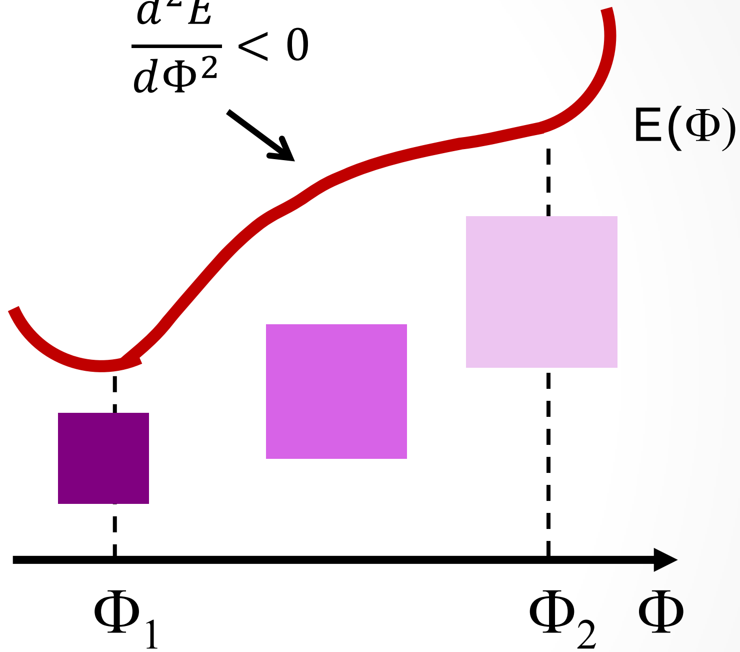
1) Transitions in the crust: 3D density space

- (n,p,e) matter:
- $\Phi = \{\delta n_q(k)\}$ q=n,p,e
- $\frac{d^2E}{d\Phi^2} < 0 \Rightarrow$

$$C(k) = \det \frac{\partial^2 E}{\partial \delta n_{ij}} < 0$$



energy density



- density or volume (order parameter) -

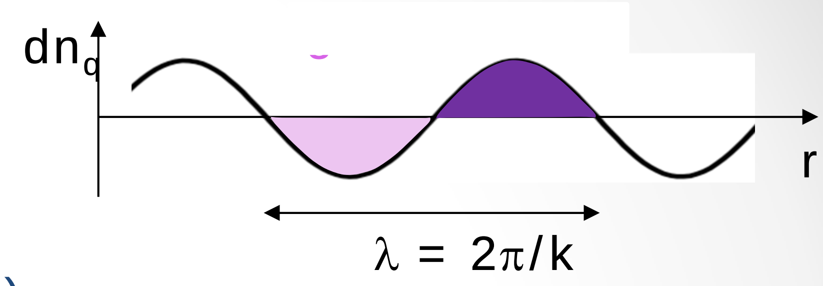
Crust-core phase transition

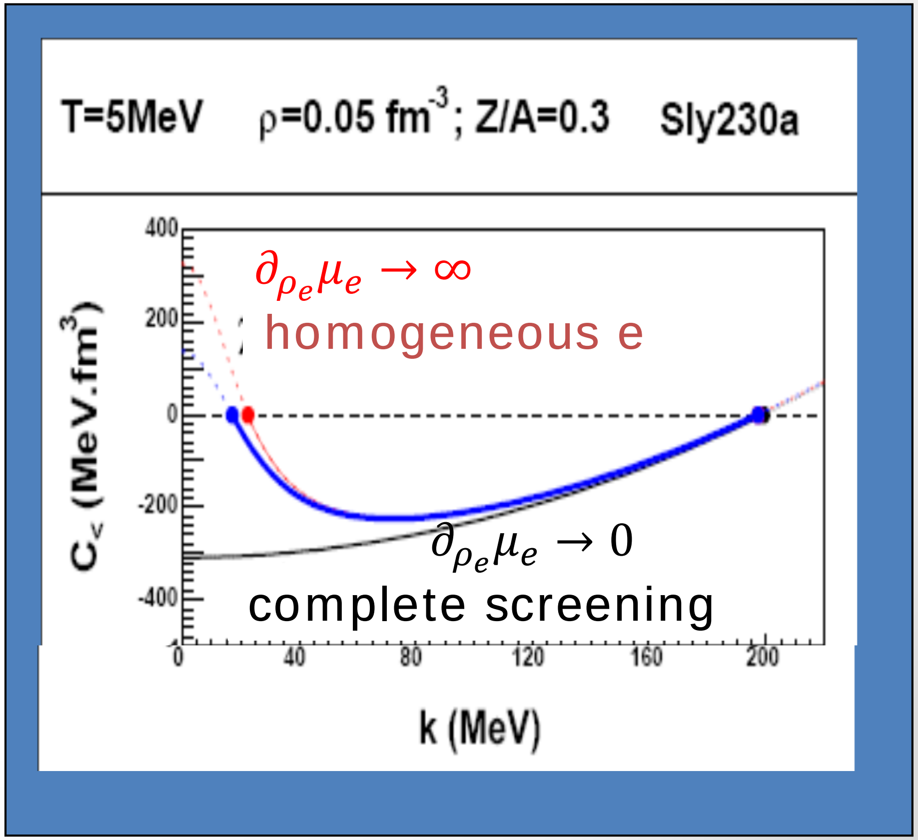
$$C(k) = \det \frac{\partial^2 E}{\partial \delta n_{ij}} < 0$$

$$C^f_{NMe} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p & 0 \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p & 0 \\ 0 & 0 & \partial_{\rho_e}\mu_e \end{pmatrix} \quad \text{Response to thermal (k=0) fluct.}$$

$$+ \begin{pmatrix} C^f_{nn} & C^f_{np} & 0 \\ C^f_{pn} & C^f_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} k^2 \quad \text{Surface term}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & -\alpha \\ 0 & -\alpha & \alpha \end{pmatrix} \frac{1}{k^2} \quad \text{Coulomb term}$$





Stellar matter at $\rho<\rho_0$ is unstable against finite size fluctuations => **cluster formation**

2) Transitions in the core : different dof, but still 3D

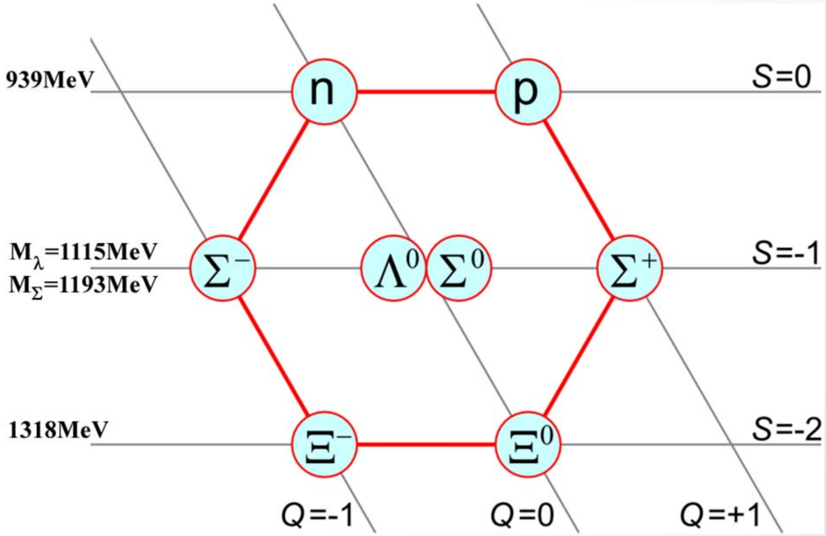
a. Hadronic matter: the baryon octet

$$\varepsilon_{RMF} = \sum_{j=1}^{8} \frac{1}{\pi^2} \int_0^{p_{Fj}} dp\, p^2 \sqrt{p^2 + m_j^{*2}} + \frac{1}{2}\left(m_\sigma^2 \sigma_0^2 + m_\omega^2 \omega_0^2 + m_\rho^2 \rho_0^2\right) + \text{non-lin. terms}$$

$$\omega_0 = \frac{1}{m_\omega^2} \sum_{j=1}^{8} g_{vj} n_j \;;\; b_0 = \frac{1}{m_\rho^2} \sum_{j=1}^{8} \tau_{3j} g_{\rho j} n_j \;;\; \sigma_0 = \frac{1}{m_\sigma^2} \sum_{j=1}^{8} g_{\sigma j} \rho_{sj} \;;\; m_j^* = m_j - g_{\sigma j} \sigma_0$$

- Equilibrium of strong interactions: three densities n_Q, n_B, n_S

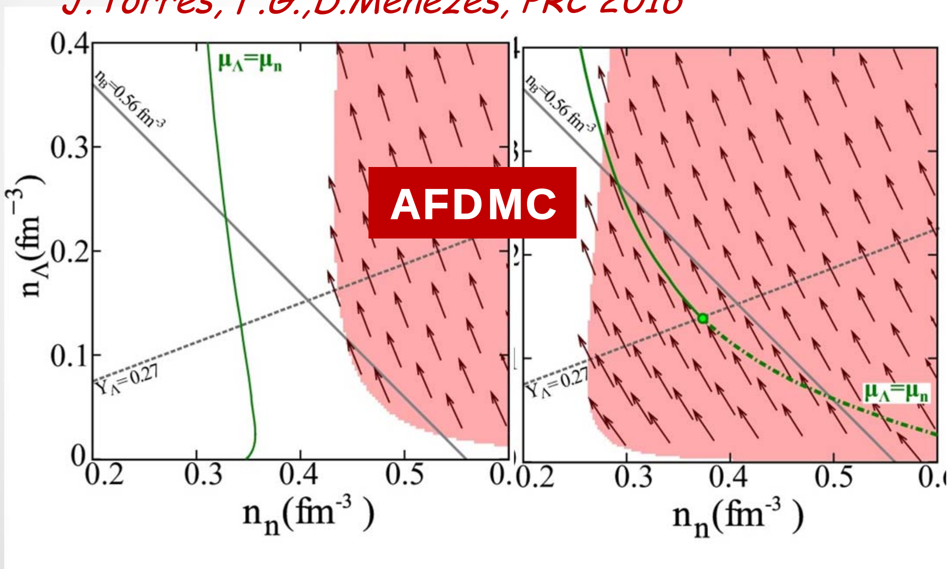
- $$\frac{d^2E}{d\Phi^2} < 0 \Rightarrow C = \det \frac{\partial^2 E}{\partial n_{ij}}$$

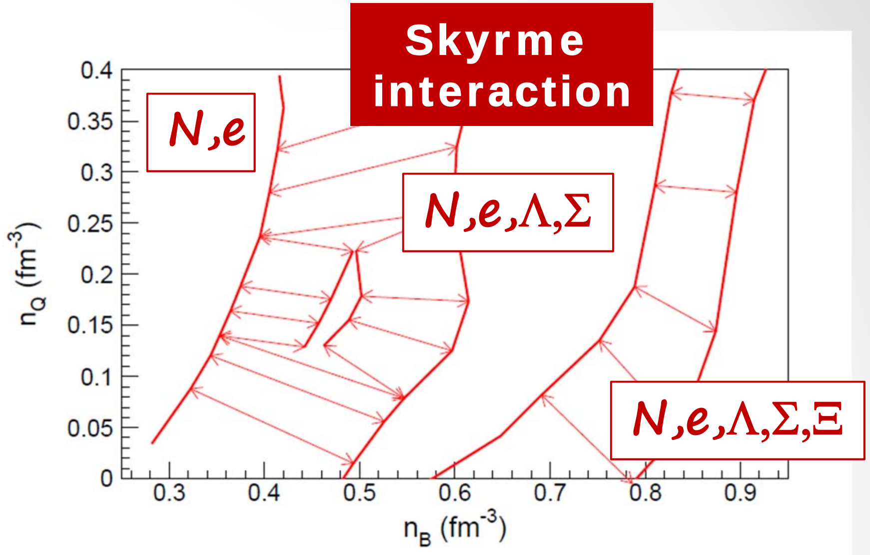


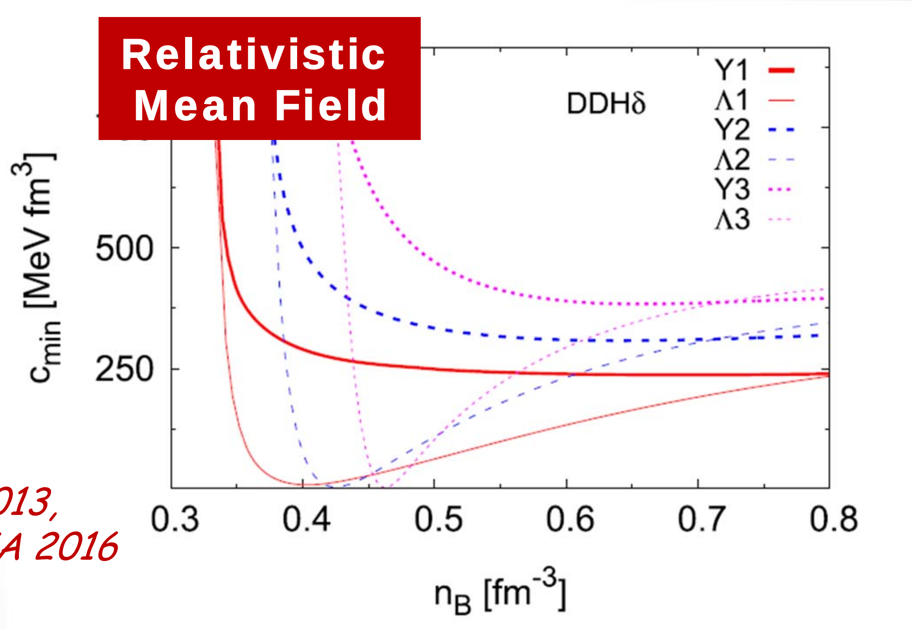
2) Transitions in the core : different dof, but still 3D

- Results are extremely model dependent

J.Torres, F.G.,D.Menezes, PRC 2016







F.G.,A.Raduta and M.Oertel, PRC 2012, PRC 2013, JPhysG 2015, EPJA 2016

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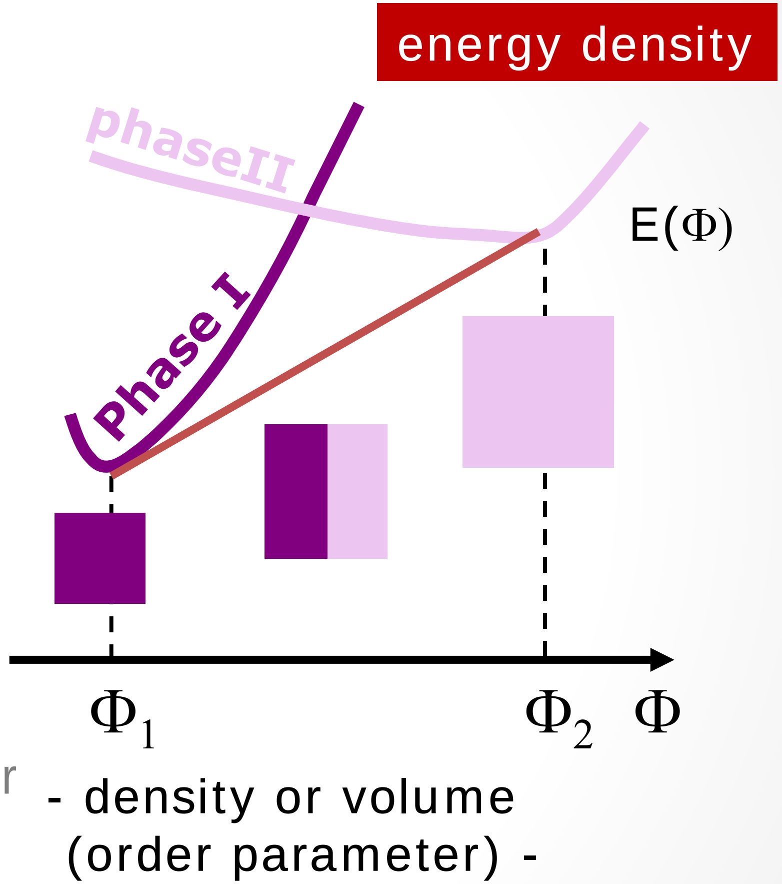
2) Transitions in the core : different dof, but still 3D

b. Deconfined matter: free quarks u,d,s => $E(n_B,n_S,n_Q)$

- No unified model for confined and deconfined matter
- Effective model (no confinement, no gluons) in the quark phase: MIT, NJL, (P)NJL, QMDD… w/wo color superconductivity (2SC, CFL phases)
- $e_{sdu}(\rho) < e_{had}(\rho)$ => hybrid star
- $e_{sdu}(\rho_{eq}) < 930$ MeV

=> Absolutely stable SQM =>quark star

- Results are extremely model dependent



- A nice collection of recent results: special issue EPJA 52 (2016)

Conclusion: three possible families of neutron stars