Dense Matter EoS and applications in Core Collapse SuperNovae and Neutron Stars

Francesca Gulminelli - LPC Caen, France

Lecture II: nuclear physics in the neutron star crust and observational consequences

1. The Wigner-Seitz cell and the outer crust
2. The physics of the inner crust
3. Extension to finite temperature
4. The impact of nuclear physics on compact stars
   a. Mass, radii => EoS parameters
   b. Crust structure => nuclear masses
   c. Pulsar glitches => crust-core transition
   d. Cooling => superfluidity and symmetry energy
   e. Core collapse => weak processes in n-rich nuclei
   f. GW emission => EoS parameters

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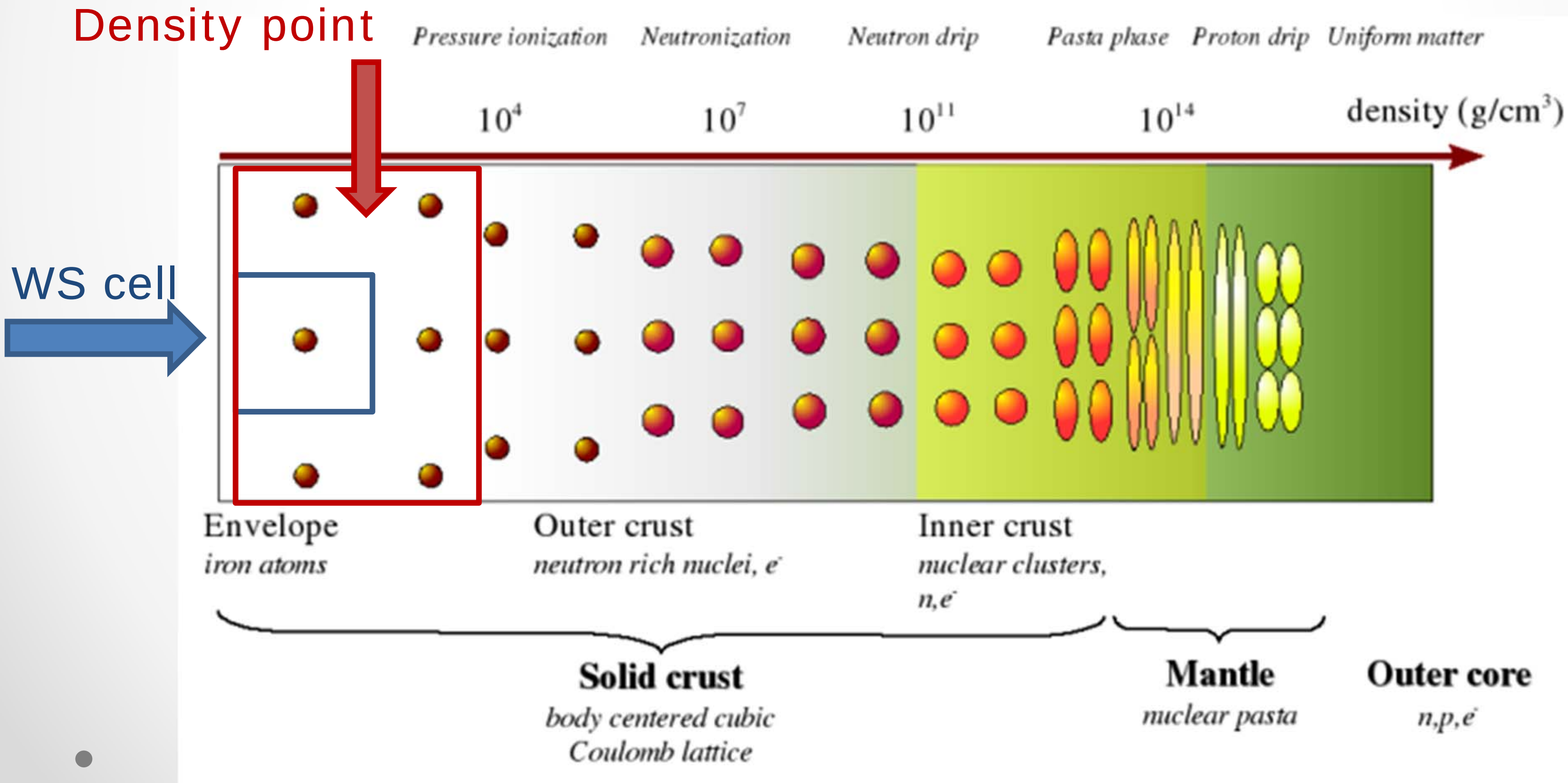
The Wigner-Seitz cell

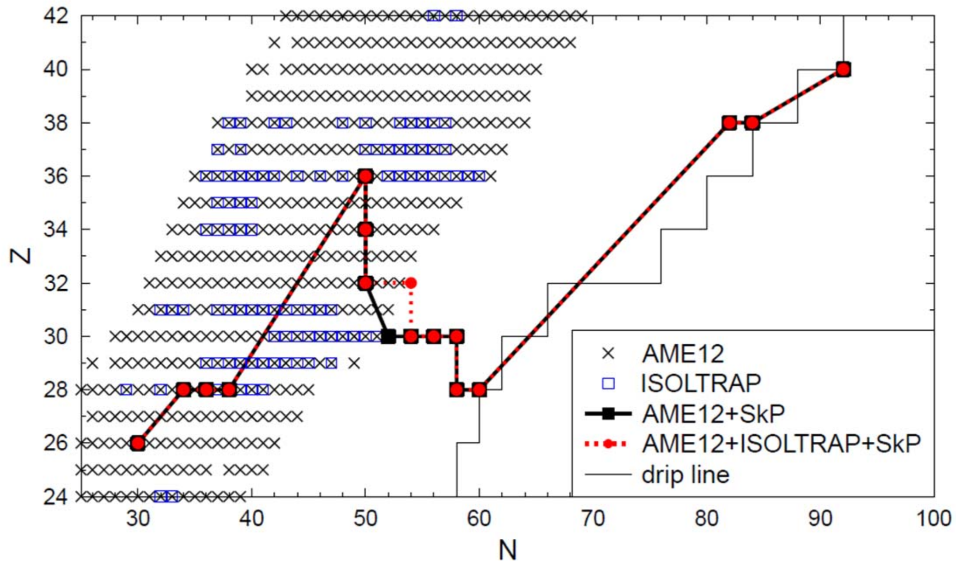
- Below saturation matter is clusterized
- At T=0: solid state: BCC lattice
- Ground state energy density: $\varepsilon(\rho) = \frac{\sum_i E_{WS}}{\sum_i V_{WS}} = \frac{E_{WS}}{V_{WS}}$ =min



Below drip: the outer crust

- $\varepsilon_{WS}(n_B) = \min_Z \left(\frac{B(N,Z)}{V_{WS}} + \varepsilon_{el}(n_e) + m_p n_p + m_n n_n + \delta\varepsilon_{coul} \right)$

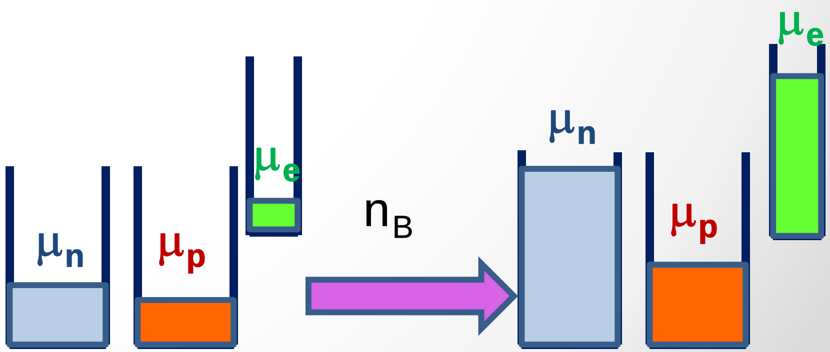
Only depends on B(N,Z) => the nuclear mass



Kreim *et al.* Int. J. Mass Spectrometry 349, 63 (2013)

Matter is increasingly n-rich for increasing density!

$\mu_n = \mu_p + \mu_e \qquad \mu_e \propto (n_e)^{1/3} = (n_p)^{1/3}$



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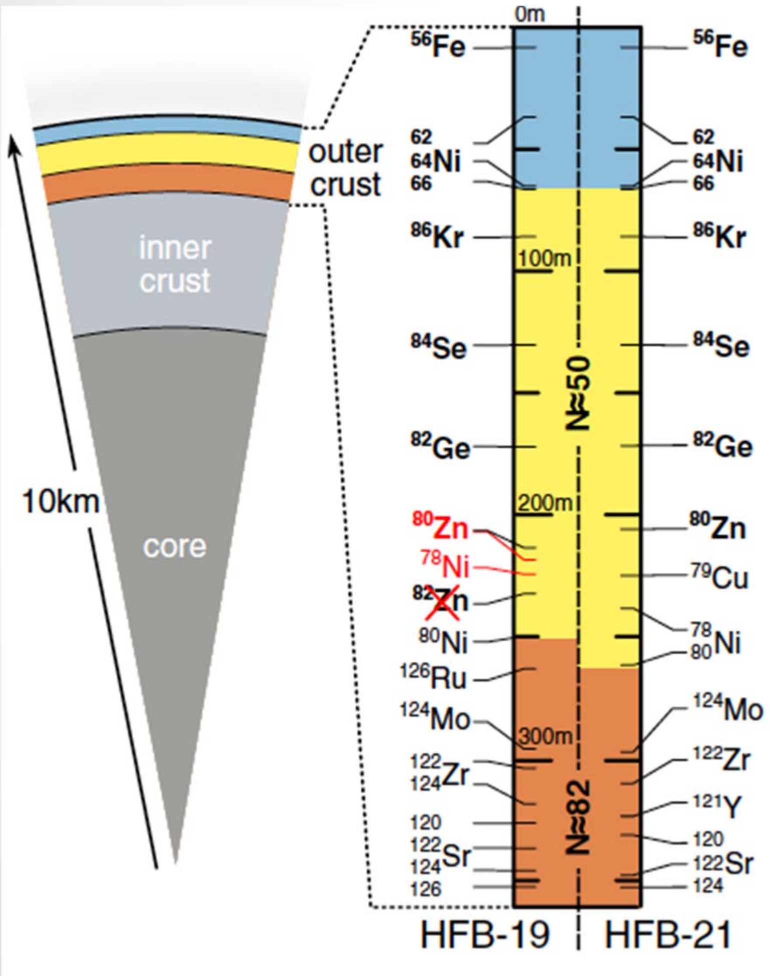
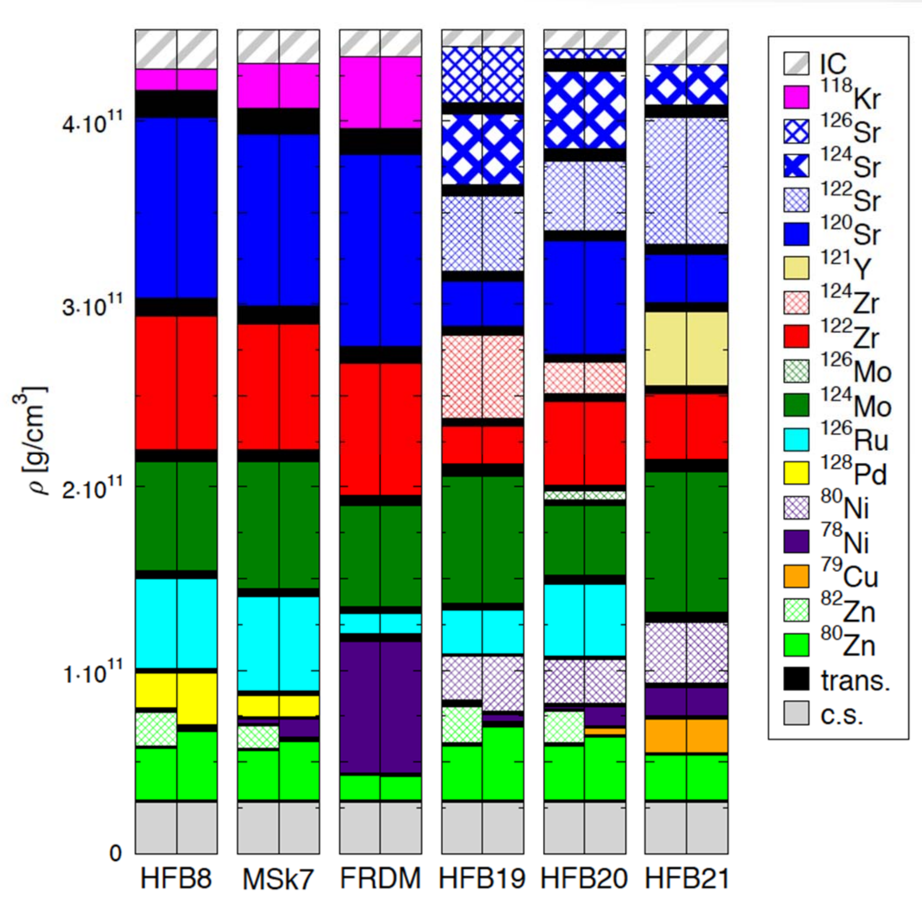


Wolf *et al.*, PRL 110, 041101 (2013)



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The inner crust

- At sufficiently high densities, neutron drip occurs => nuclear modelling cannot be avoided

- **A model for the energy density**

Exp.(or DFT) mass

Nucleus-gas interaction

$$\varepsilon_{WS} = \frac{B(N,Z)}{V_{WS}} + \varepsilon_{HM}(n_{ng}) + \varepsilon_{el}(n_e) + \delta\varepsilon_{coul} + \delta\varepsilon_{nuc}$$

Hom.matter EoS

(N, Z, V_{WS}, n_{ng})
variational variables

- **A variational problem for each (n_B)**

$$d\left(\varepsilon_{WS} - \mu_n\left(\frac{N_{WS}}{V_{WS}} - n_n\right) - \mu_p\left(\frac{Z_{WS}}{V_{WS}} - n_p\right)\right) = 0$$

- **A set of coupled equations for N, Z, V_{WS}, n_{ng}**

Pethick, Ravenhall, ARNPS 45(1995)429, G.Watanabe NPA 676 (2000) 455

Clusters in the medium

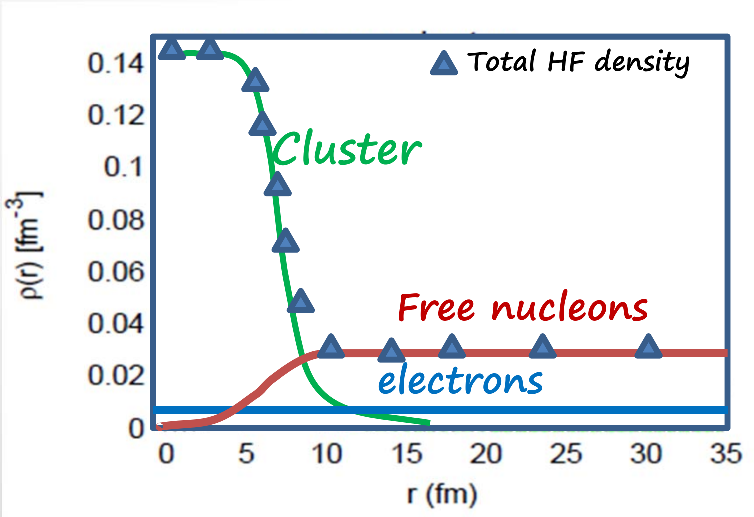
P. Papakonstantinou, et al. Phys.Rev.C 88(2013) 045805

A density fluctuation?

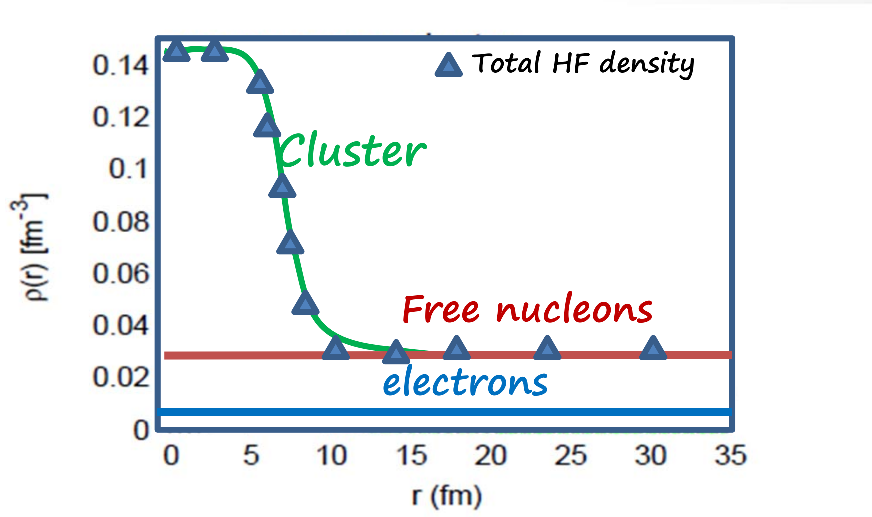
- LS, Shen, Hempel, Sumiyoshi, Mishustin, Raduta&FG…

An ensemble of bound states?

- Roepke,Typel, FG



r-cluster



e-cluster

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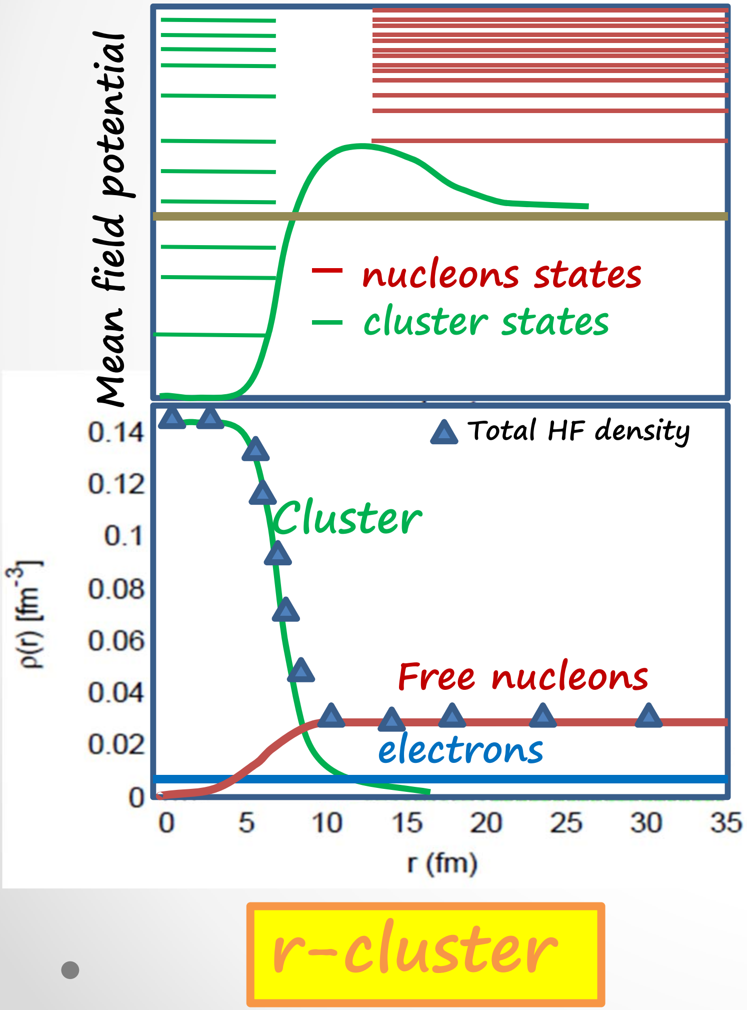
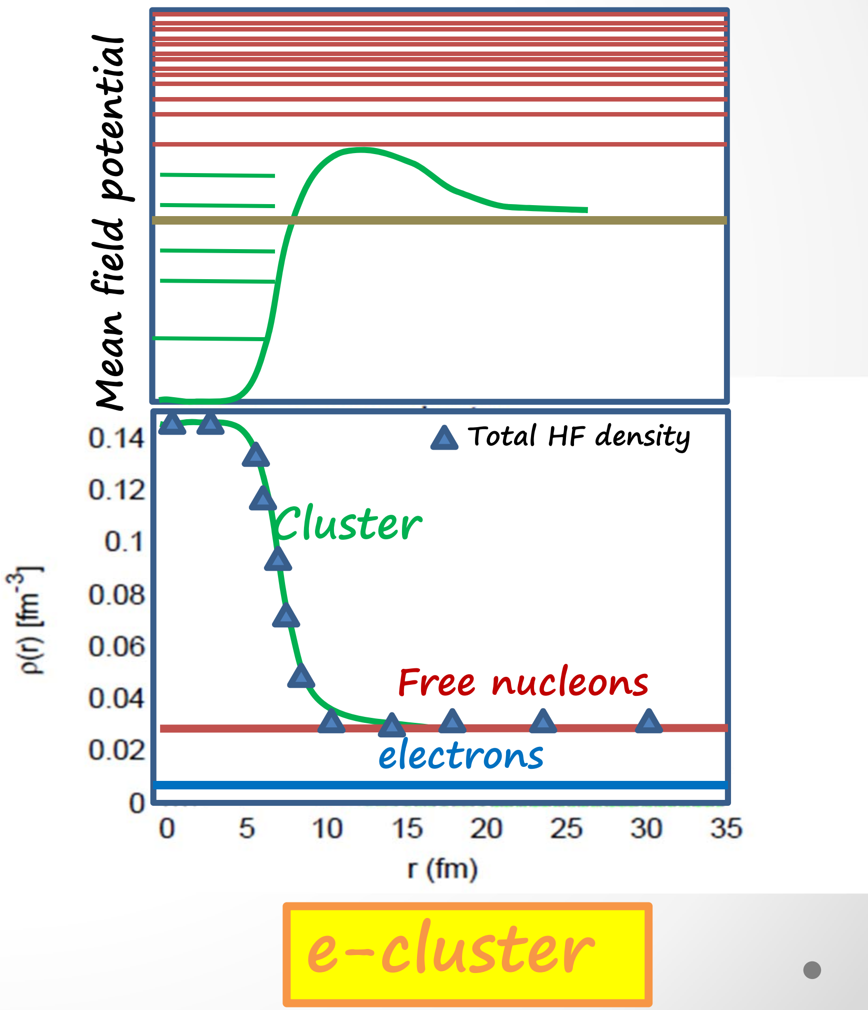


An ensemble of bound states?



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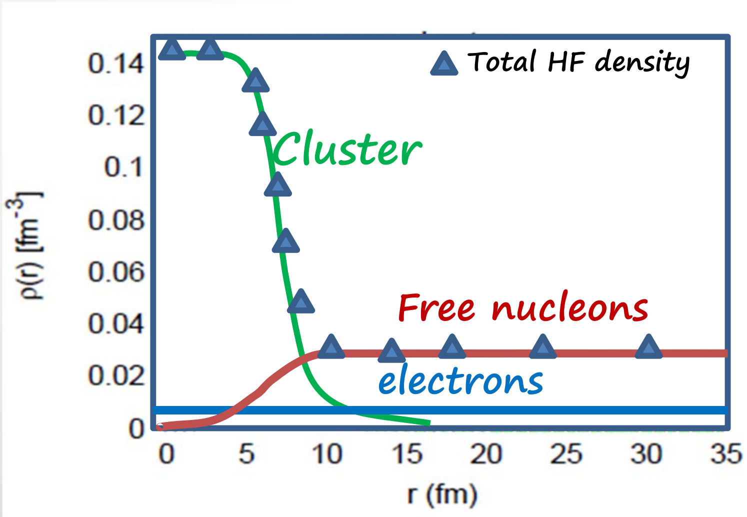
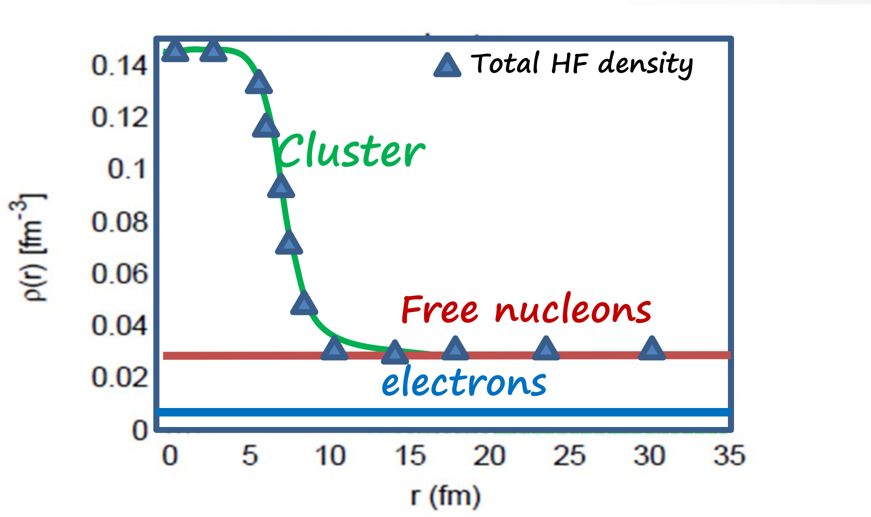
The two descriptions correspond to the same density profile
⇒ They can be mapped to produce the same energy



r-cluster



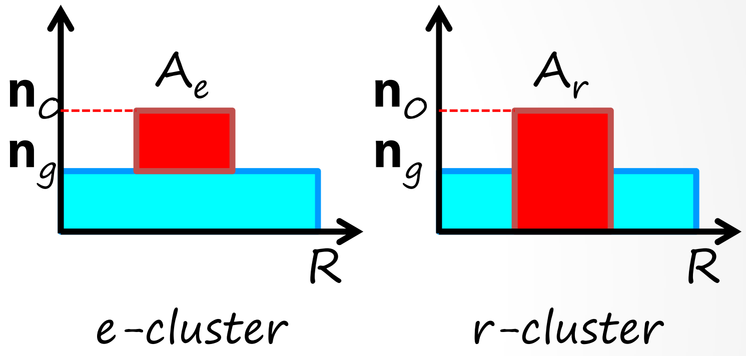
e-cluster

Schematic mapping

- $A_e = A_r\left(1 - \frac{n_g}{n_0}\right)$
- $A_{WS} = \begin{cases} A_e + n_g V_{WS} \\ A_r + n_g(V_{WS} - V_{cl}) \end{cases}$
- $E_{WS} = \begin{cases} E_0 + \varepsilon_g V_{WS} + \delta E_e \\ E_0 + \varepsilon_g(V_{WS} - V_{cl}) + \delta E_r \end{cases}$

$=> \delta E_r - \varepsilon_g V_{cl} = \delta E_e$

No isospin for simplicity

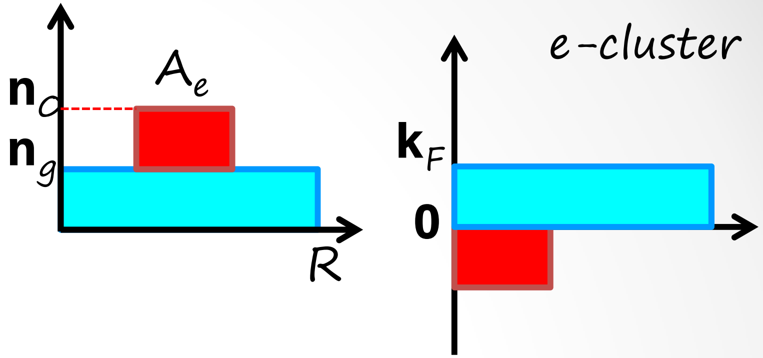


=> The in-medium binding energy shift in the r-representation can be mapped to the e-representation and vice-versa

In-medium effects

- **Pauli-blocking shifts**

(G.Roepke PRC79(2009))
The high-k states are occupied by the gas => they cannot be occupied by the bound quasi-particles



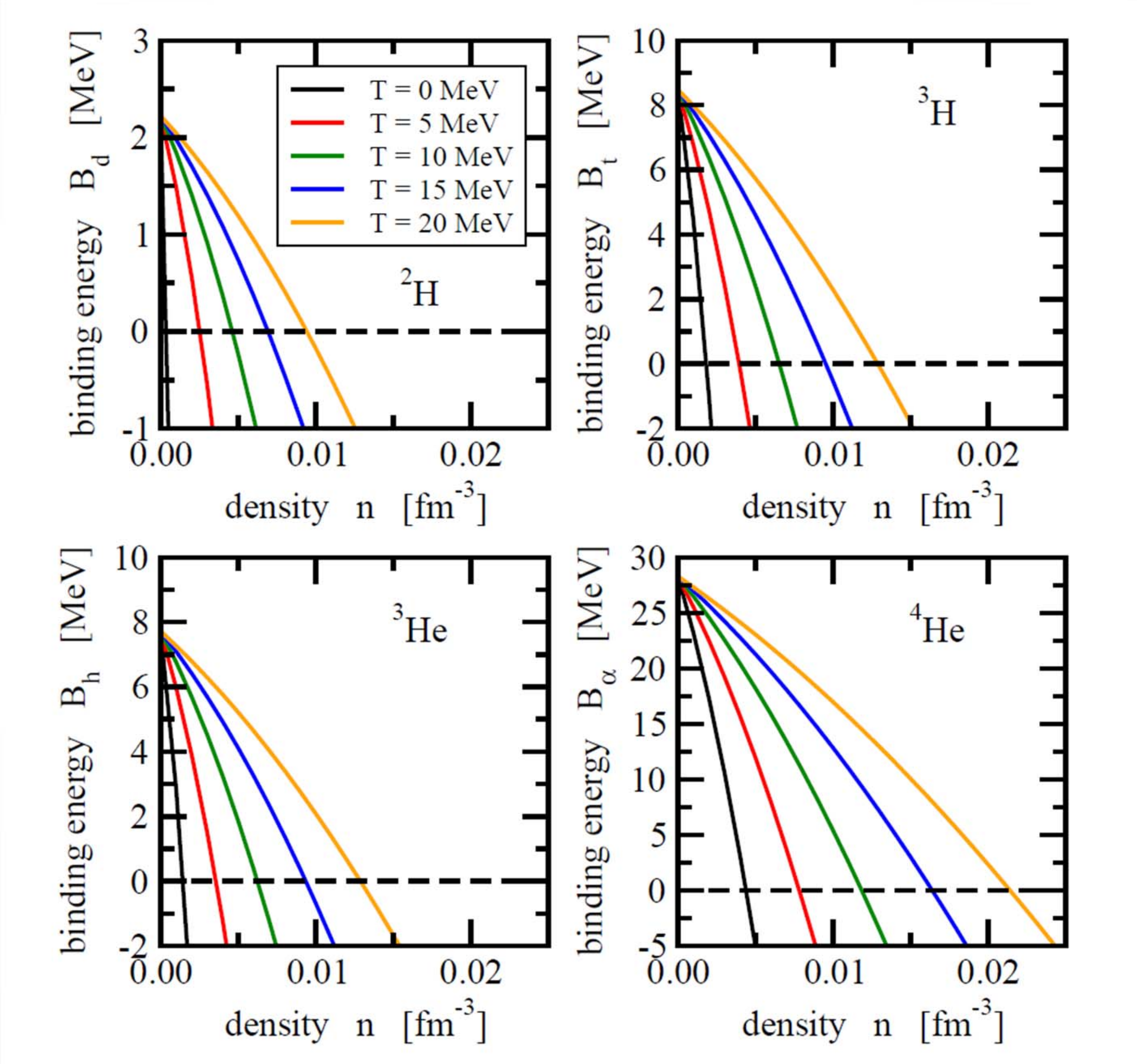
$$\sum_{i=1}^{A} e_i \varphi(1,..,A) - \sum_{i',j'} \sum_{j<i} (1 - f_i - f_j) V_{iji'j'} \varphi(1',..,A') = 0$$

- Exemple: deuteron

$$\delta E_e = \frac{1}{N} \sum_q \varphi_d^*(q) \left[f_n \left(\frac{P}{2} + q \right) + f_p \left(\frac{P}{2} - q \right) \right] V(q,q') \varphi_d(q')$$

- Jastrow wave function+separable interaction => analytical results up to A=4

Mott density



S.Typel et al, NPA 2009

In-medium effects

- **Thomas-Fermi shifts**

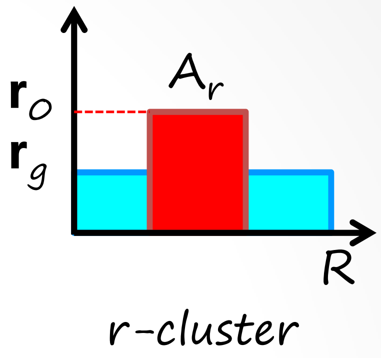
In the local density approximation, the in-medium correction to r-clusters is only a surface effect

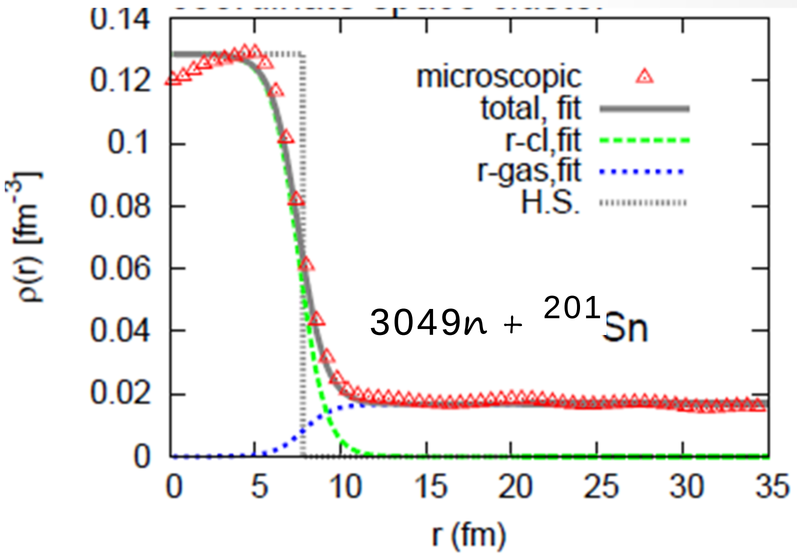
$\delta E_r = \delta E_{surf}$

$$\delta E_{surf} = \int_0^{R_{WS}} d^3 r \varepsilon[\rho(r)] - \varepsilon(\rho_0)\frac{A_r}{\rho_0} - \varepsilon_g V_{WS}$$

=> the mapping allows determining the bulk effect due to the Pauli-blocking mechanism

$$\delta E_e = -\varepsilon_g \frac{A_r}{\rho_0} + \delta E_{surf}$$



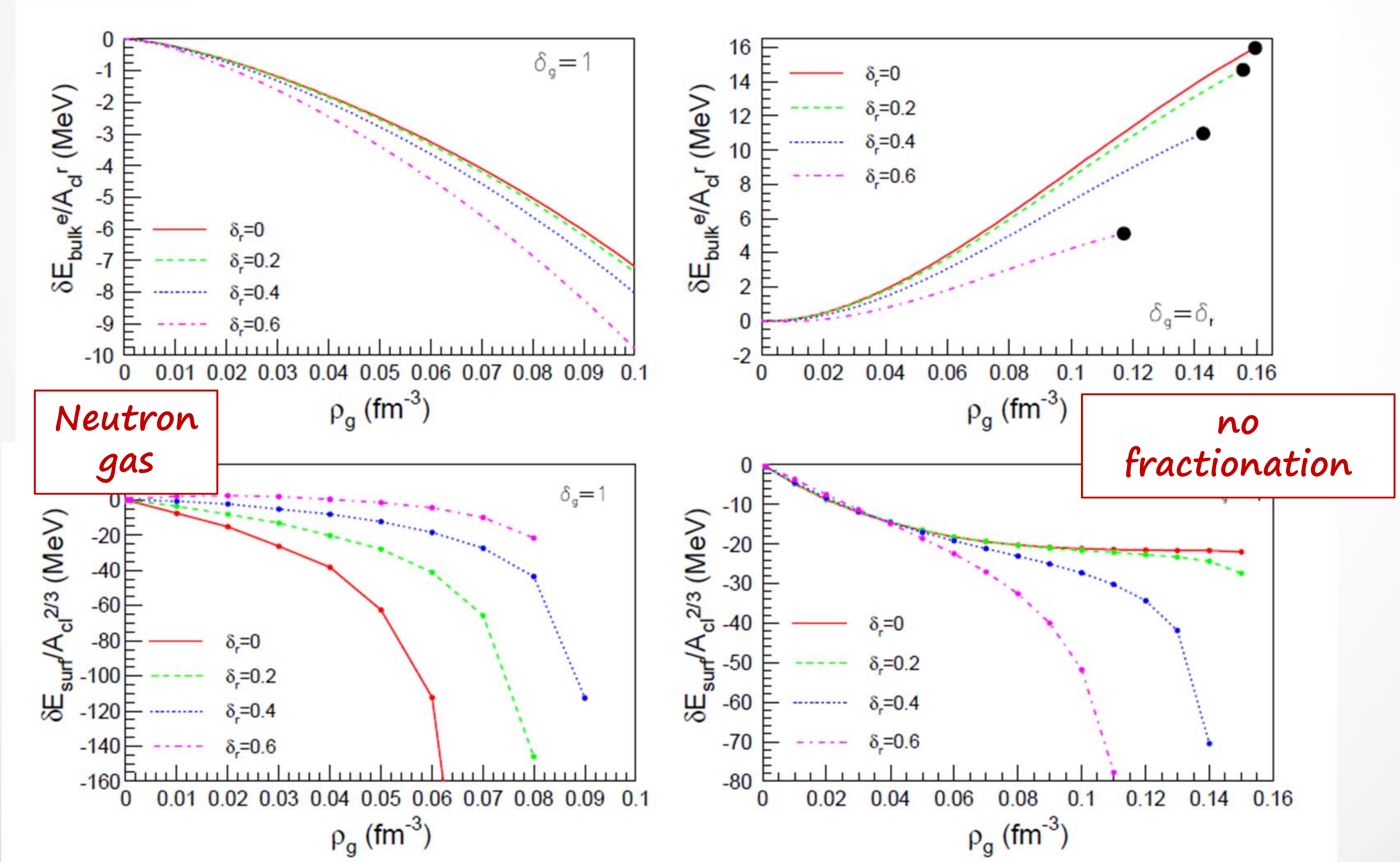


Thomas-Fermi shifts

F.Aymard et al, EPJA 2015

Sly4 Skyrme functional

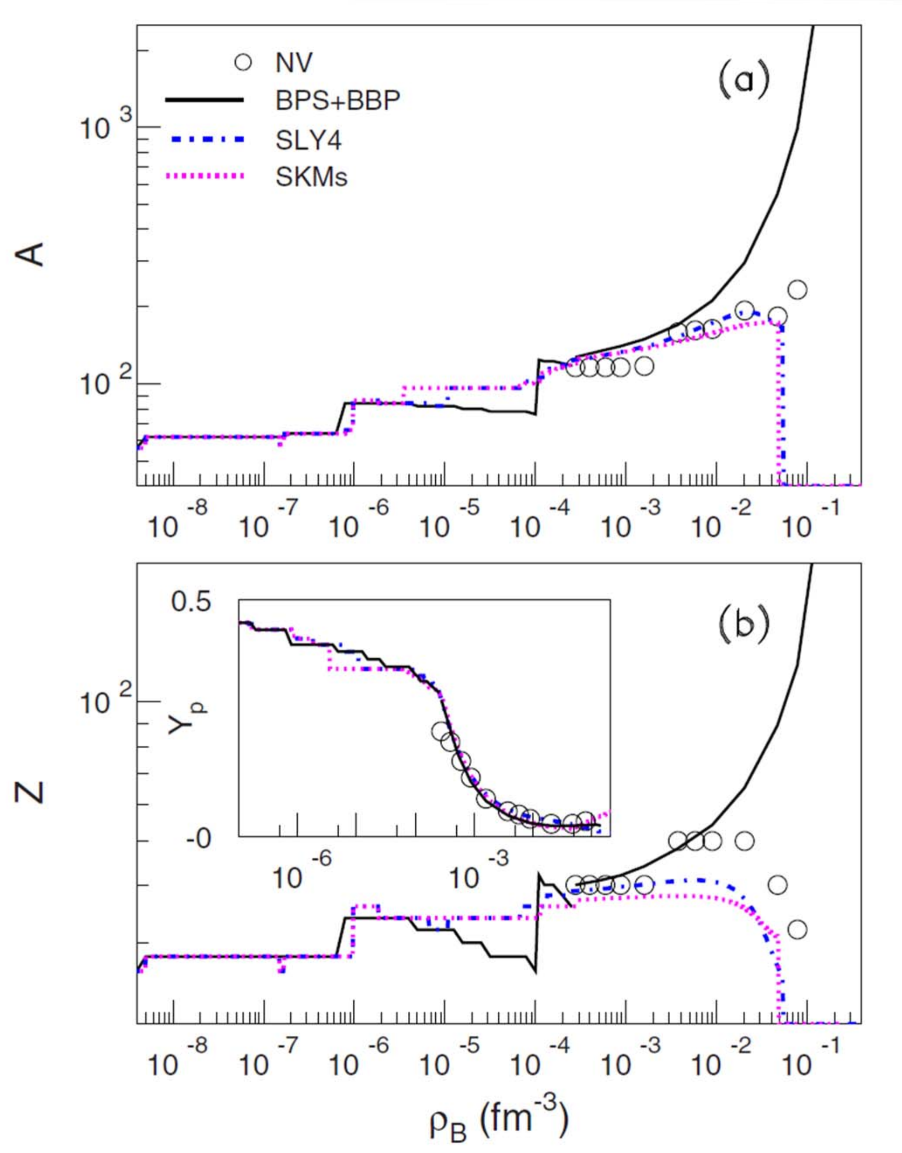
$$\delta = \frac{(\rho_{0n} - \rho_{0p})}{\rho_0}$$



Composition of the neutron star crust

NV: *HF in the WS cell* (Negele&Vautherin)
BPS+BBP: *LDM* (Baym et al.)
Sly4
SKM*

- Melting of nuclei in the dense medium
- (Small) effect of the effective interaction
- Shell effects important even after drip



- F.G., A.Raduta PRC 2016

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T>0: the Single Nucleus Approximation

- **A model for the free energy density**

$$f_{WS} = \frac{B(N,Z) - TS(N,Z)}{V_{WS}} + f_{HM}(n_{ng}, n_{pg}, T) + f_{el}(n_e, T) + \delta f_{coul} + \delta f_{nuc}$$

$(N, Z, V_{WS}, n_{ng}, n_{pg})$ variational variables

- **A variational problem for each (n_B, T)**

$$d\left(f_{WS} - \mu_n\left(\frac{N_{WS}}{V_{WS}} - n_n\right) - \mu_p\left(\frac{Z_{WS}}{V_{WS}} - n_p\right)\right) = 0$$

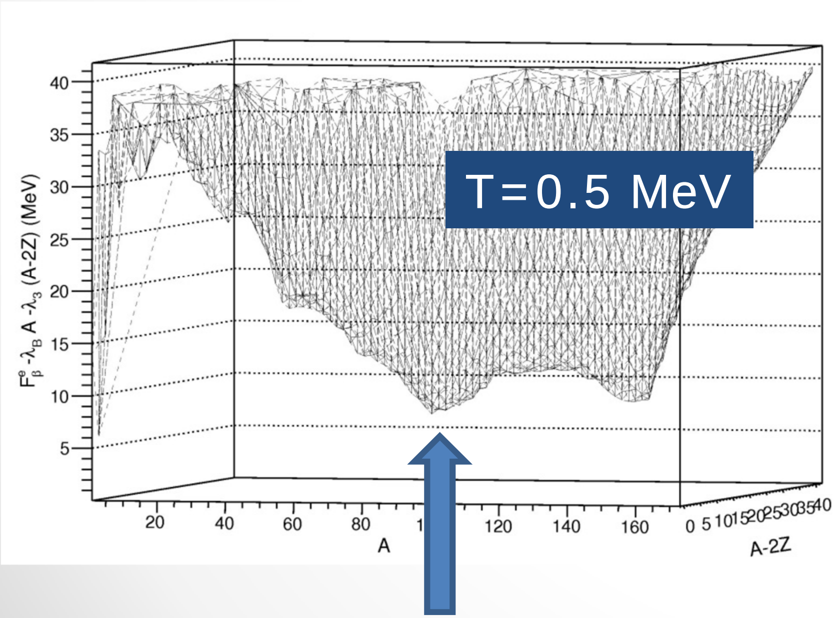
- **A set of coupled equations for $N, Z, V_{WS}, n_{ng}, n_{pg}$**

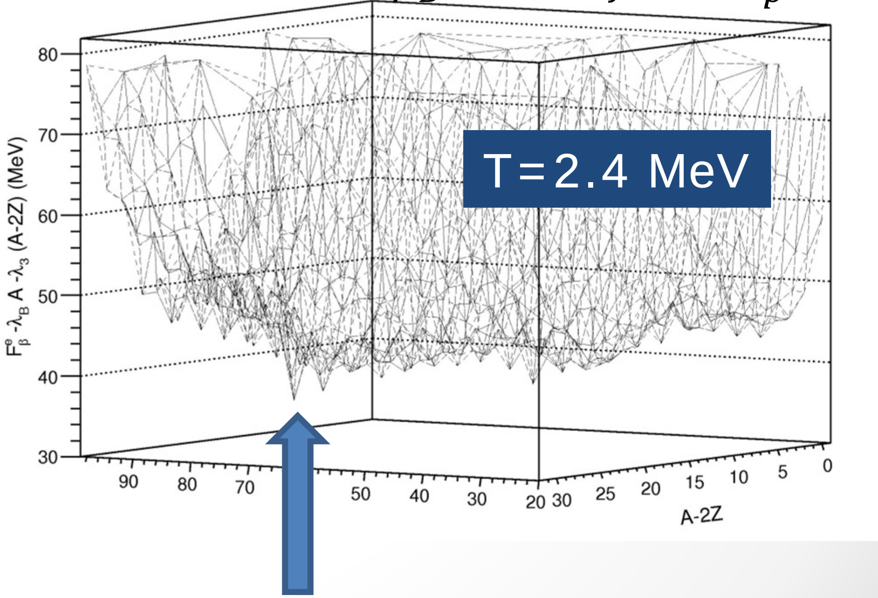
J. M. Lattimer and F. D. Swesty, NPA 535, 331 (1991).
H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi,NPA 637, 435 (1998).

T>0: the Single Nucleus Approximation

- It is the standard strategy for supernova simulations
- Still, it is a very poor treatment of the finite temperature problem.

$\rho_B = 10^{-3} fm^{-3}, Y_p = 0.39$





The absolute minimum is not representative of the free energy landscape

T>0: beyond the SN approximation

- One WS cell

$$d\left(f_{WS}-\mu_n\left(\frac{N_{WS}}{V_{WS}}-n_n\right)-\mu_p\left(\frac{Z_{WS}}{V_{WS}}-n_p\right)\right)=0$$

N,Z,n_{gn},n_{gp} variational variables linked by the strict conservation law in the cell

- Many WS cells

f_{tot}

$$\mathrm{d}\left(T\sum_k p_k ln p_k+\left(E_{tot}-\langle\hat{H}\rangle_V\right)-\mu_n\left(N_{tot}-\langle\hat{N}\rangle_V\right)-\mu_p\left(Z_{tot}-\langle\hat{Z}\rangle_V\right)\right)=0$$

$$k=\left\{n_i^{(k)},N_i,Z_i\; i=1,\dots,N_k;N_{free}^{(k)}Z_{free}^{(k)}\right\}$$

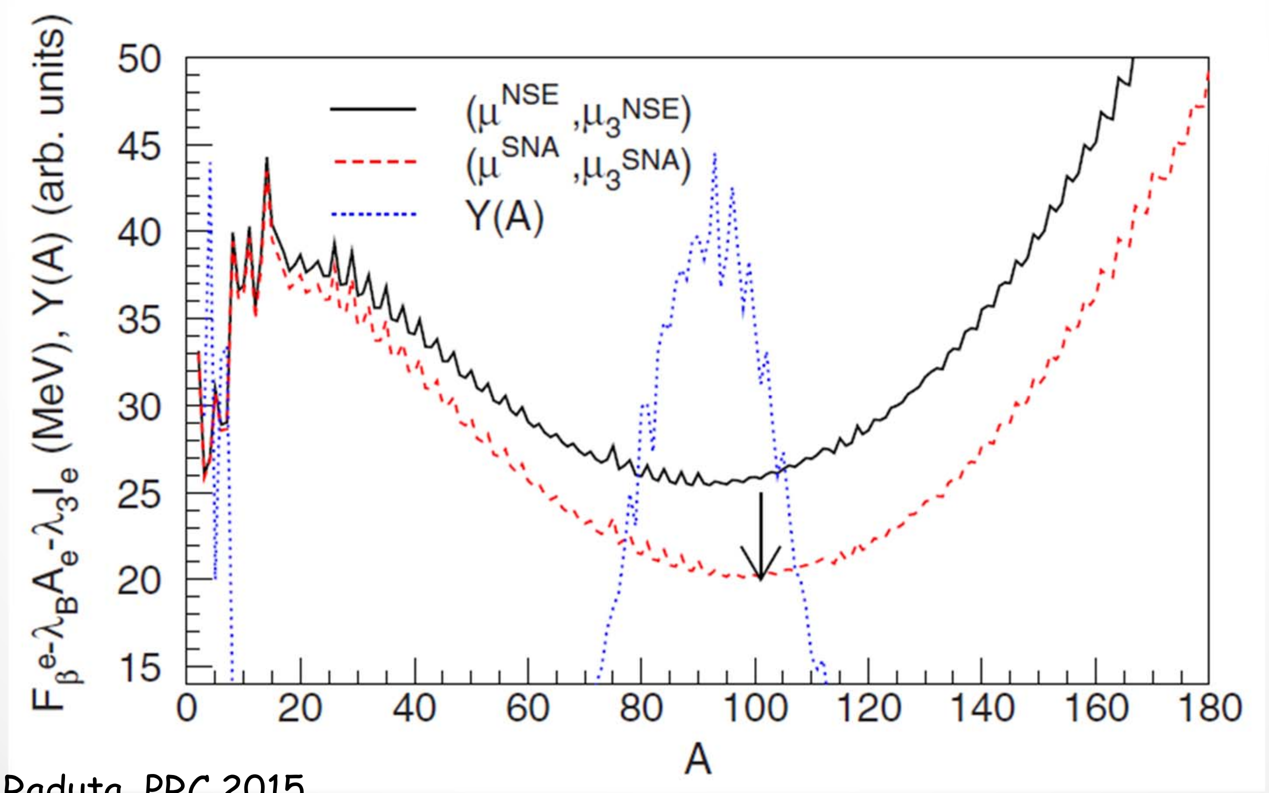
$$\langle n_{NZ}\rangle=exp-\beta\left(B_m-TS-\mu_n N-\mu_p Z\right)$$

N,Z variational variables linked by the loose conservation law in the cell through the global chemical potential

- Different equations at T>0
- Same ground state T=0 solution

NSE versus SNA

- The Single Nucleus Approximation is valid as long as average values are concerned
- Still, deviations increase with density and temperature



F.Gulminelli, A.Raduta, PRC 2015

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A-Mass and radius: the TOV equation

Gravitational force on the shell per unit area:

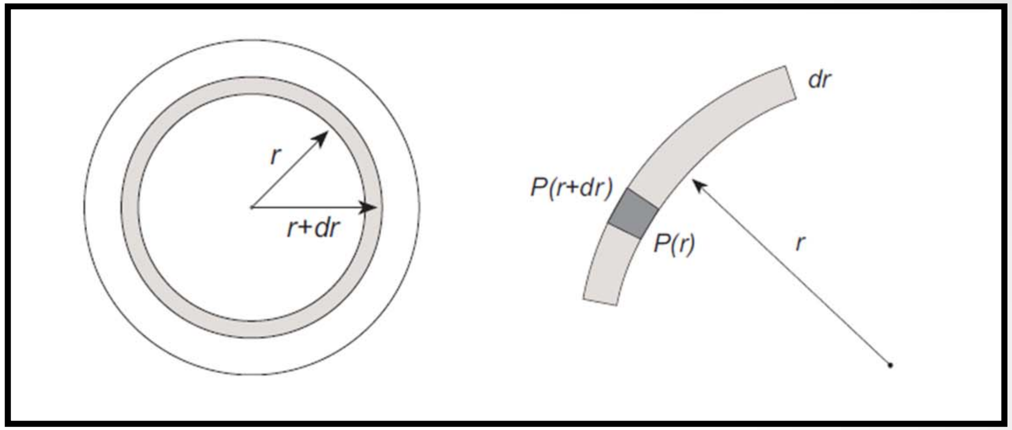
$$F_g = -dmg = -\varrho g dr$$

Pressure gradient through dr:

$$P(r) - P(r+dr) = -\frac{dP}{dr}dr = F_p$$

Hydrostatic equilibrium:

$$F_g + F_p = 0 \Rightarrow \boxed{\frac{dP}{dr} = -\varrho g = -\varrho \frac{Gm}{r^2}}$$



Mass-density relation

$$m(r) = \int_0^r dr 4\pi r^2 \rho(r) \Rightarrow \boxed{\frac{dm}{dr} = 4\pi r^2 \rho(r)}$$

Tolman-Oppenheimer-Volkoff: from general relativity

$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}$$

Mass and radius: the TOV equation

$$\begin{cases} \dfrac{dm}{dr} = 4\pi r^2 \rho(r) \\ \dfrac{dP(r)}{dr} = -\dfrac{G}{r^2}\left[\rho(r) + \dfrac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \dfrac{P(r)}{c^2}\right]\left[1 - \dfrac{2GM(r)}{c^2 r}\right]^{-1} \end{cases}$$

TOV equation

$m(r=0)=0$

$\rho(r=0)= \rho_c$

$P(r=0)$ from EoS

Boundary conditions

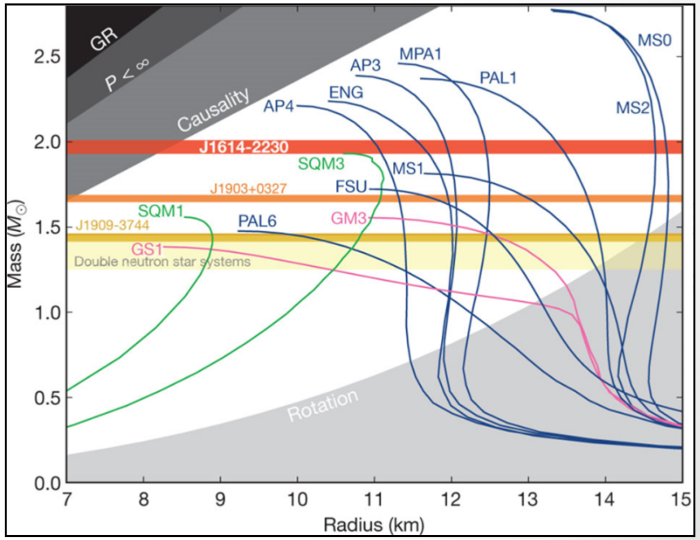
$$m(r+dr) = m(r) + dr\frac{dm}{dr}$$

$$P(r+dr) = P(r) + dr\frac{dP}{dr}$$

$\rho(r+dr)$ from EoS

Numerical solution

$$if\ P < \varepsilon\ stop\ \forall \rho_c, R = R_{max}, M = M(R_{max})$$



=> The M(R) curve depends only on the EoS

Constraining the model parameters

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

$$x = \frac{\rho - \rho_0}{\rho_0}$$

$$e = \frac{\varepsilon_B}{\rho}$$

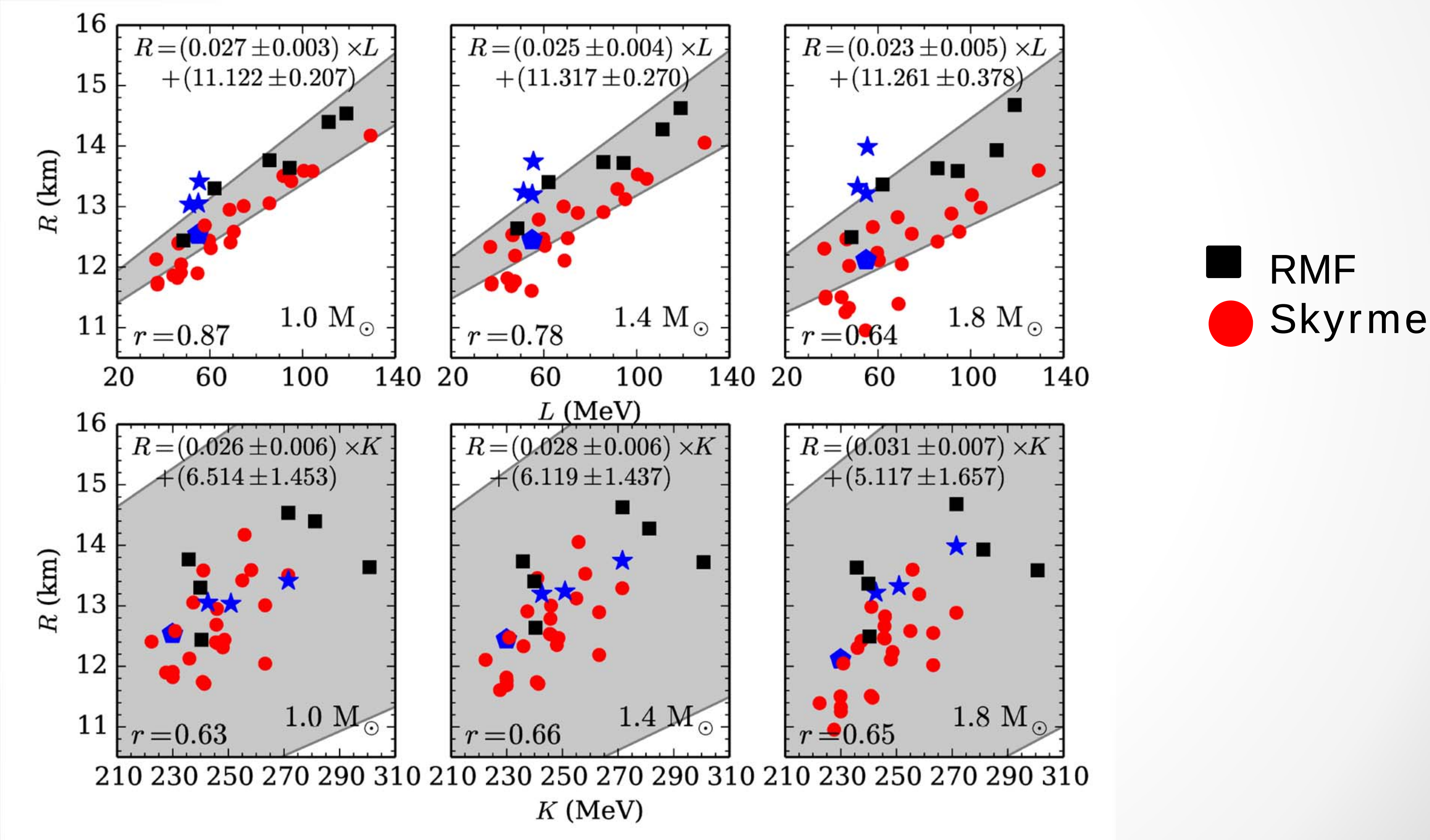
- Definition of empirical parameters
  - Any EoS can be Taylor expanded

$$e(\rho, \delta) = e_{IS}(\rho) + e_{IV}(\rho)\delta^2 + O(\delta^3)$$

$$= \left(\boldsymbol{E_0} + \frac{1}{18}\boldsymbol{K_0}x^2 + O(x^3)\right) + \left(\boldsymbol{J_0} + \frac{1}{3}\boldsymbol{L}x + \frac{1}{18}\boldsymbol{K_{sym}}x^2 + O(x^3)\right)\delta^2$$

$$\boldsymbol{p = \rho^2 \frac{de}{d\rho}}$$

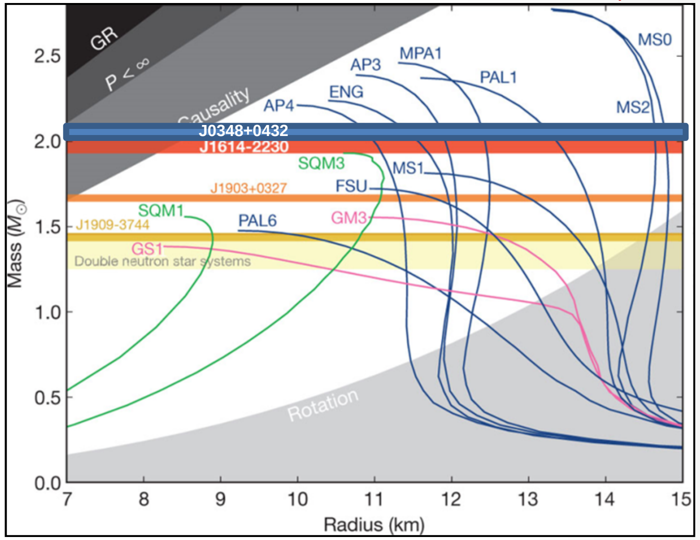
NS radius and the EoS



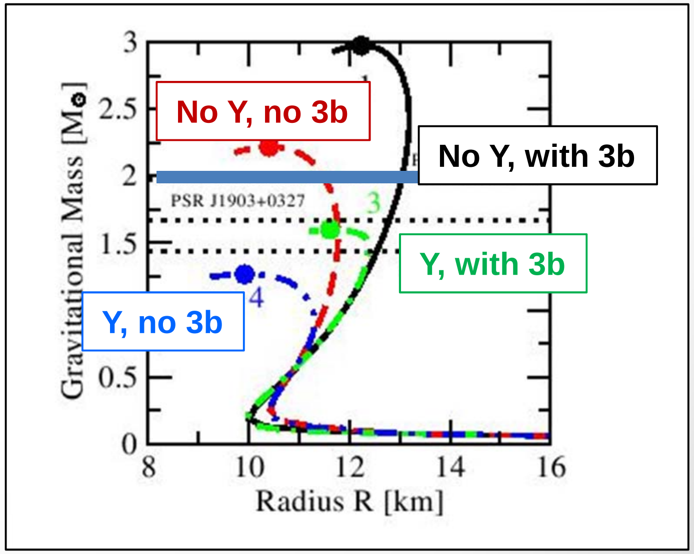
M.Fortin et al, ArXiv 2016

NS mass and the hyperon puzzle

P. Demorest et al., Nature 467 1081 (2010).
J. Antoniadis et al., Science, 340, 6131 (2013).



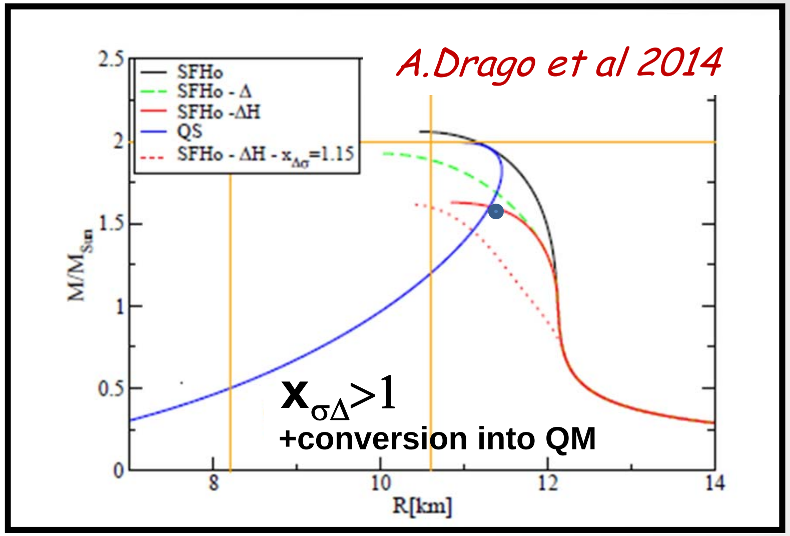
- The highest mass is associated to the highest central density.
- If $\mu(\rho) > m_Y c^2 + U_Y$, hyperon Y should appear
- The appearence of a new degree of freedom softens the EoS=>reduces the mass
- 2M$_O$ neutron star should not exist if U_Y is calculated with microscopic BHF based on experimental bare interactions

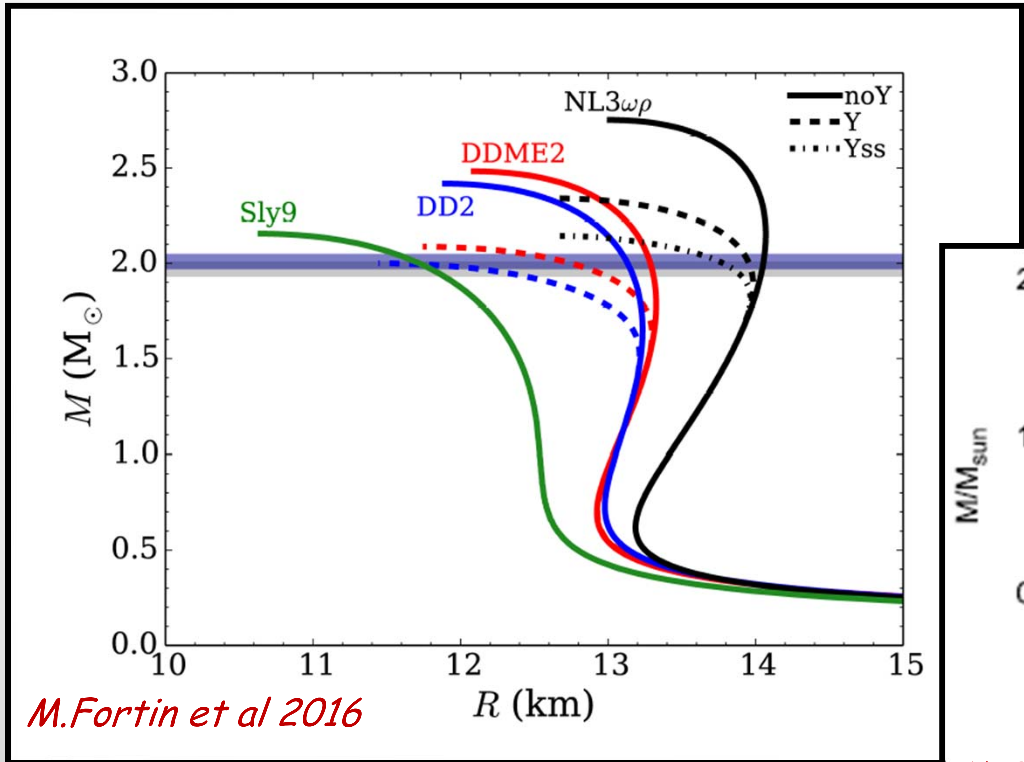


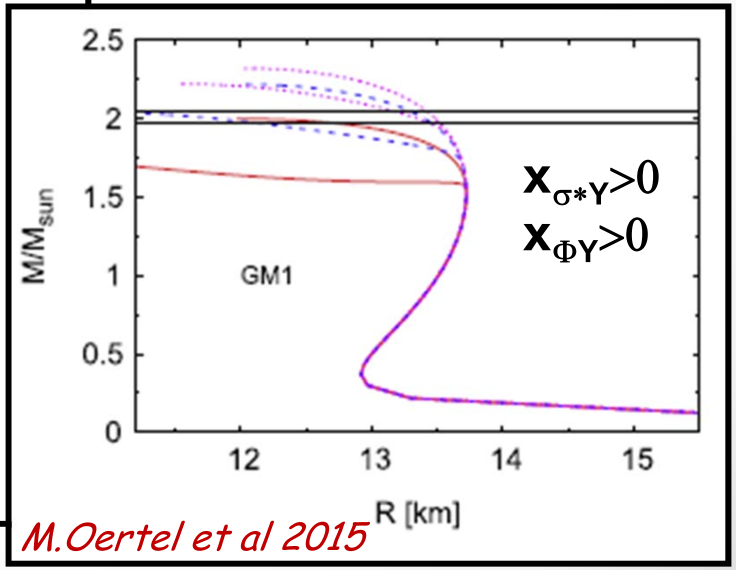
- *I.Vidana et al, Europhys.Lett.94:11002,2011*

The hyperon puzzle: solutions

- Stiffening of the EoS above rho0?
- New strangeness couplings at high density ?
- Transition to quark matter ?







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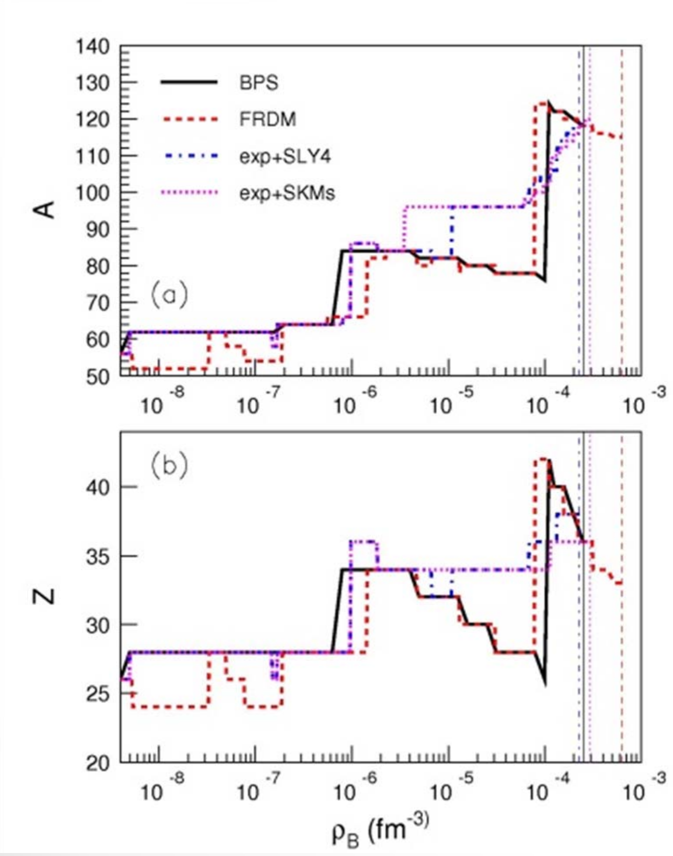
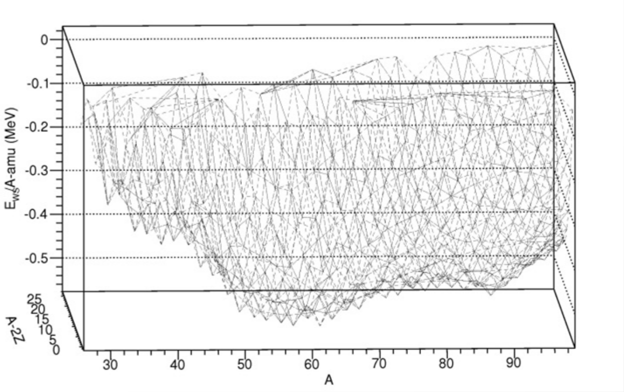
B-The importance of mass measurements



FRDM: *up to date LDM* (Moller&Nix)
BPS: *LDM Myers-Swiatecki* (Baym et al.)
Sly4
SKM*
This work with exp data when available

Many quasi-degenerate minima!



F.Gulminelli, A.Raduta, ArXiV:1504.04493

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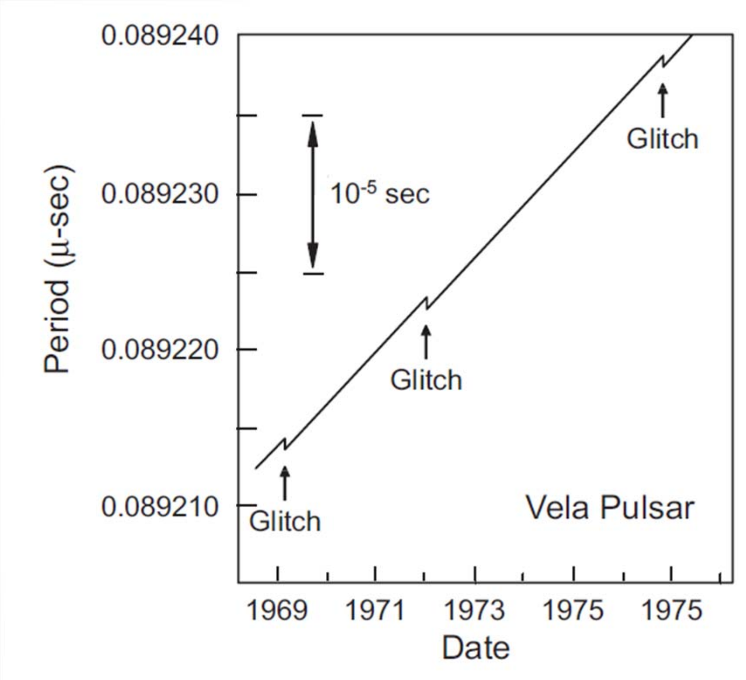
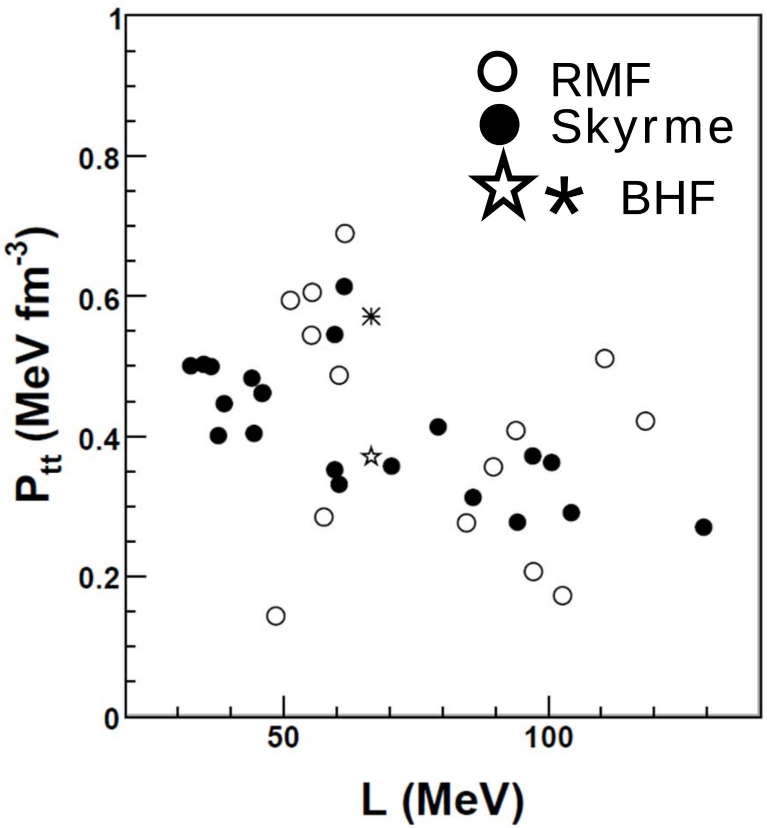
C-Pulsar glitches



- In some pulsars "glitches" are observed where the spin rate suddenly jumps to a higher value
- Glitches indicate some internal rearrangement has altered the rotation rate by a small amount.
- Sudden unpinning of the superfluid vortices from the crystal lattice during the slowing down due to the differential rotation between the fast vortices and the slow star
- Angular momentum transfer to the star which spins up

C-Pulsar glitches



C.Ducoin et al, PRC 2012

- $I_c/I>0.07$ to explain Vela data

$$I \equiv \frac{J}{\Omega} = \frac{8\pi}{3}\int_0^R r^4 e^{-\nu(r)} \frac{\bar{\omega}(r)}{\Omega} \frac{\left(\mathcal{E}(r)+P(r)\right)}{\sqrt{1-2GM(r)/r}} dr$$

$$I_{\rm crust} = \frac{8\pi}{3}\int_{R_t}^R r^4 e^{-\nu(r)} \widetilde{\omega}(r) \frac{\left(\mathcal{E}(r)+P(r)\right)}{\sqrt{1-2GM(r)/r}} dr \ .$$

$\Rightarrow P_t>0.5$ MeV fm^{-3}

Results are extremely model dependent

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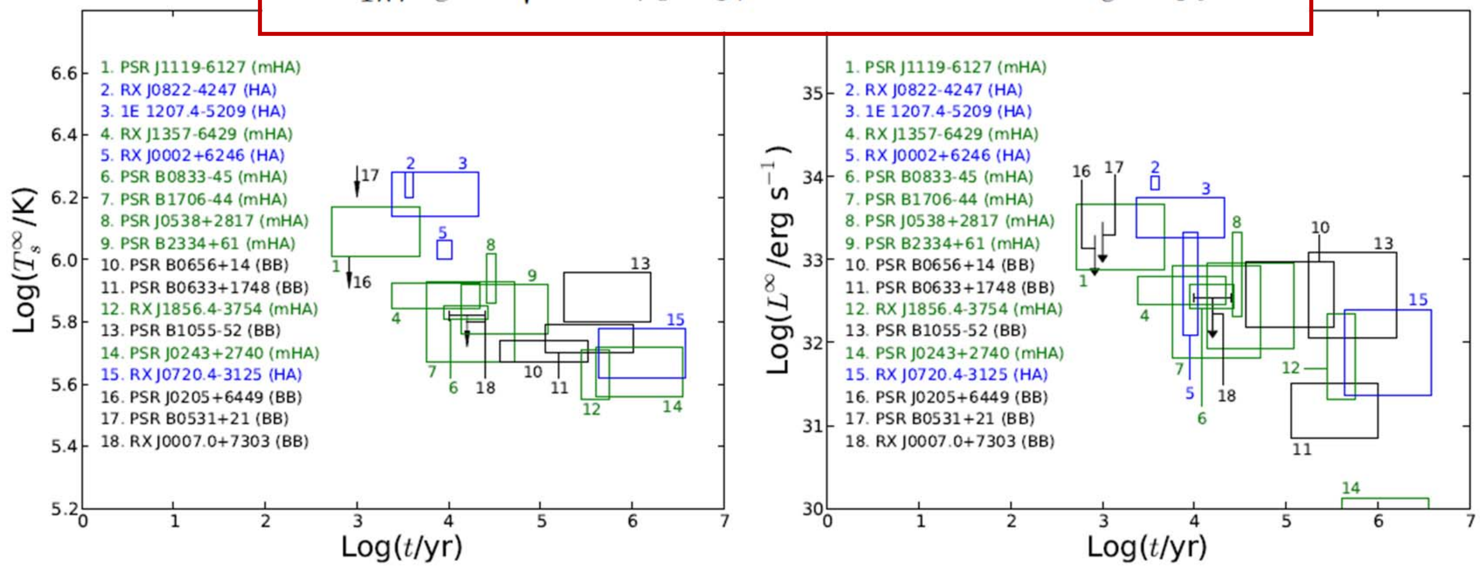
   f. GW emission => EoS parameters

D-Neutron star cooling

- Neutron stars are born hot, and cool down via neutrino emission
- Cooling curve can be inferred from luminosity measurements via atmosphere modelling
- Cooling depends on the neutrino emissivity and the heat capacity

$$\frac{L_r}{4\pi\kappa r^2} = -\sqrt{1-\frac{2Gm}{rc^2}}\,e^{-\Phi_g}\frac{\partial}{\partial r}\left(Te^{\Phi_g}\right) ,$$

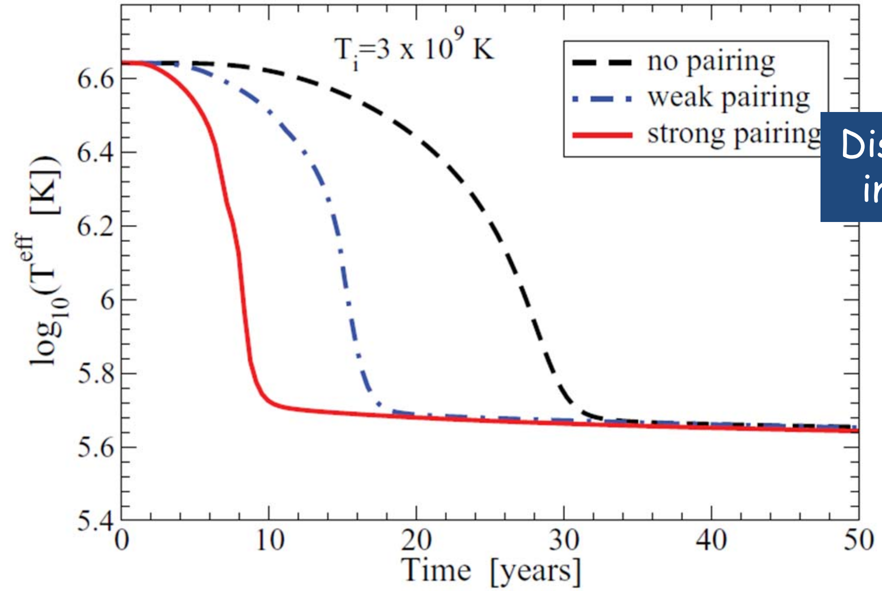
$$\frac{1}{4\pi r^2 e^{2\Phi_g}}\sqrt{1-\frac{2Gm}{rc^2}}\,\frac{\partial}{\partial r}\left(e^{2\Phi_g}L_r\right) = -Q_\nu - \frac{C_v}{e^{\Phi_g}}\frac{\partial T}{\partial t} ,$$



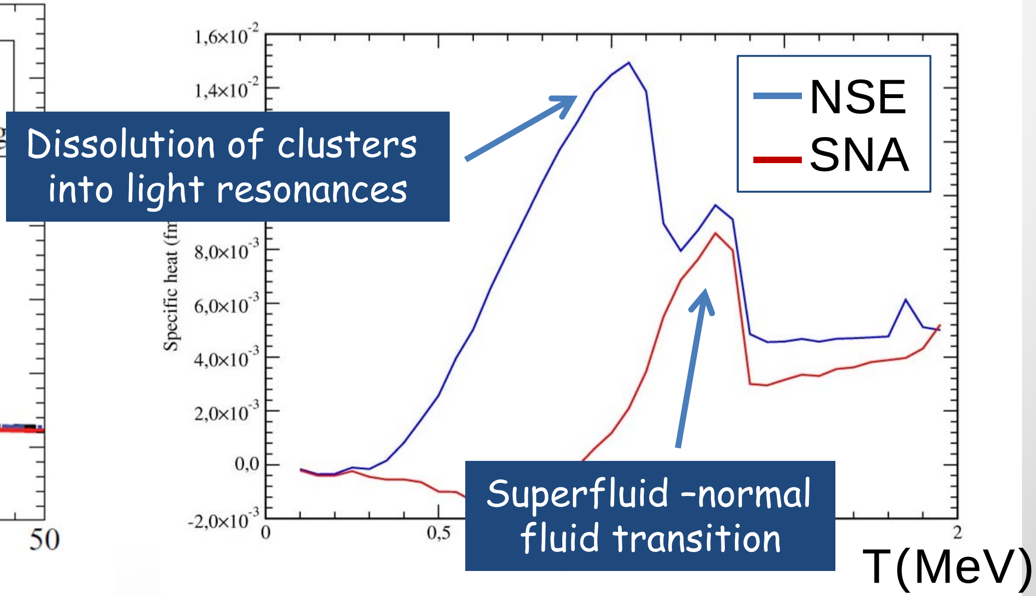
D-Neutron star cooling

- The cooling of very hot proto-neutron star is dominated by the heat capacity of the crust
- $^1S_0$ n- superfluidity is the key ingredient, but in-medium effects on the cluster distributions can play a role
- Unfortunately observations are not available

J.Margueron 2005



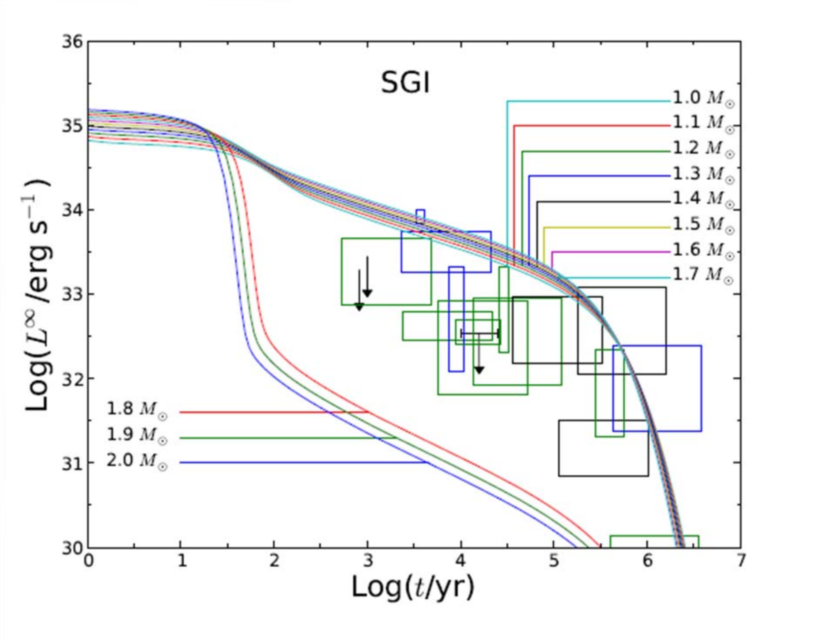
S.Burrello 2016



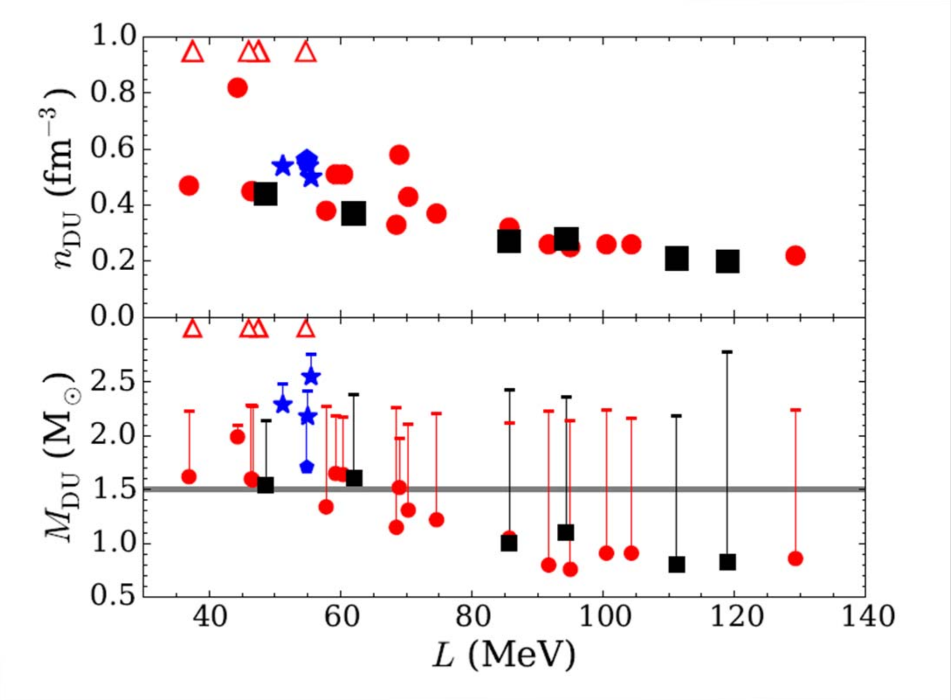
D-Neutron star cooling

- The ν emissivity dramatically depends on the possibility of DURCA: $n \rightarrow p + e + \nu$ and $p + e \rightarrow n + \nu$
- Momentum conservation implies $p_{Fn}(\rho) \leq p_{Fp}(\rho) + p_{Fe}(\rho)$ => needs a minimum proton fraction
- The minimum proton fraction allowing DURCA is determined by the EoS
- Enhanced cooling seems excluded, but the subject is still under debate

Lim 2015



Fortin 2016



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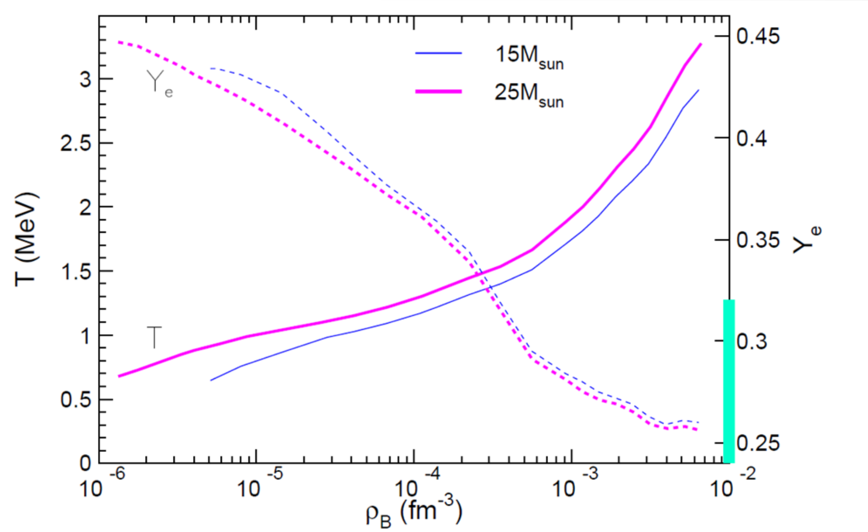
   f. GW emission => EoS parameters

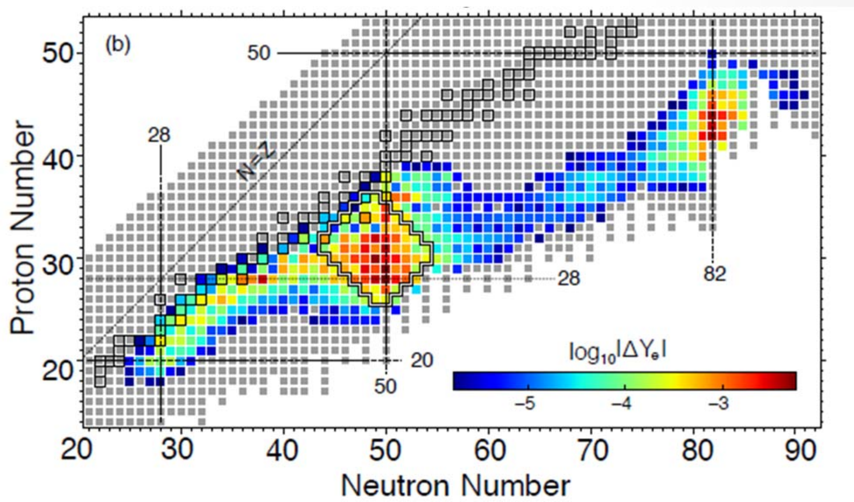
E-Electron capture and core collapse

- Matter becomes increasingly neutron-rich during core-collapse because of electron capture on nucleons and nuclei

- In the late stage of the collapse, matter is essentially constituted by exotic nuclei around the N=50 and N=82 magic numbers



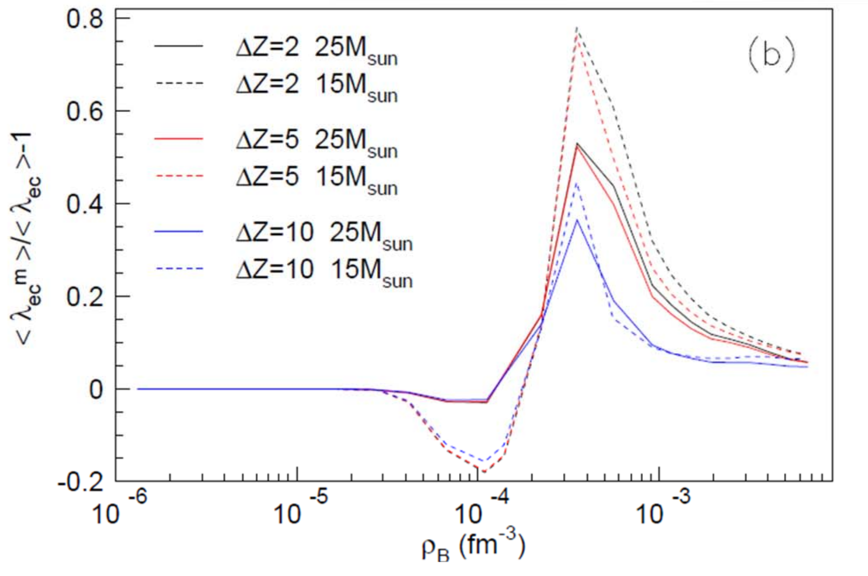
A.Raduta 2015

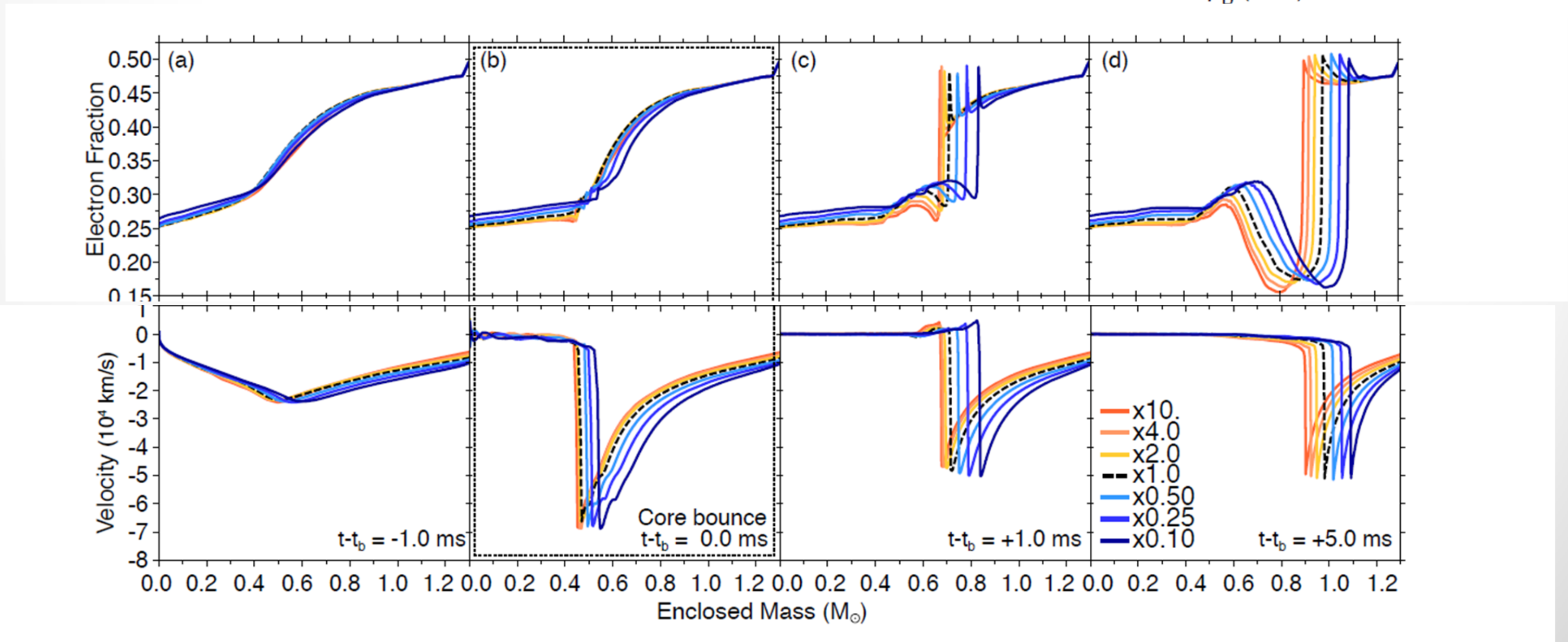


C.Sullivan 2015

E-Electron capture and core collapse

- The supernova evolution crucially depends on the e-capture rate.
- In turn, this depends on the mass of exotic N=50 nuclei and their β-decay properties.





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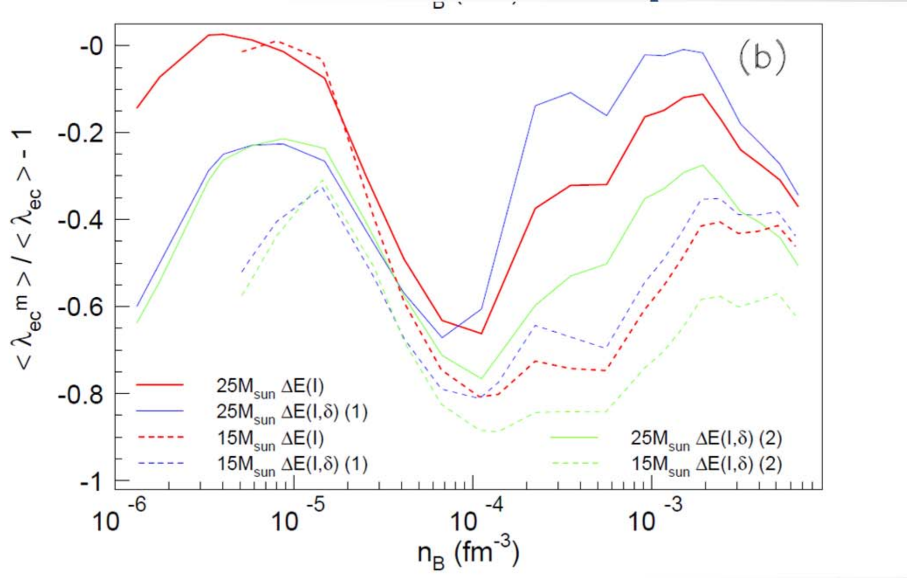

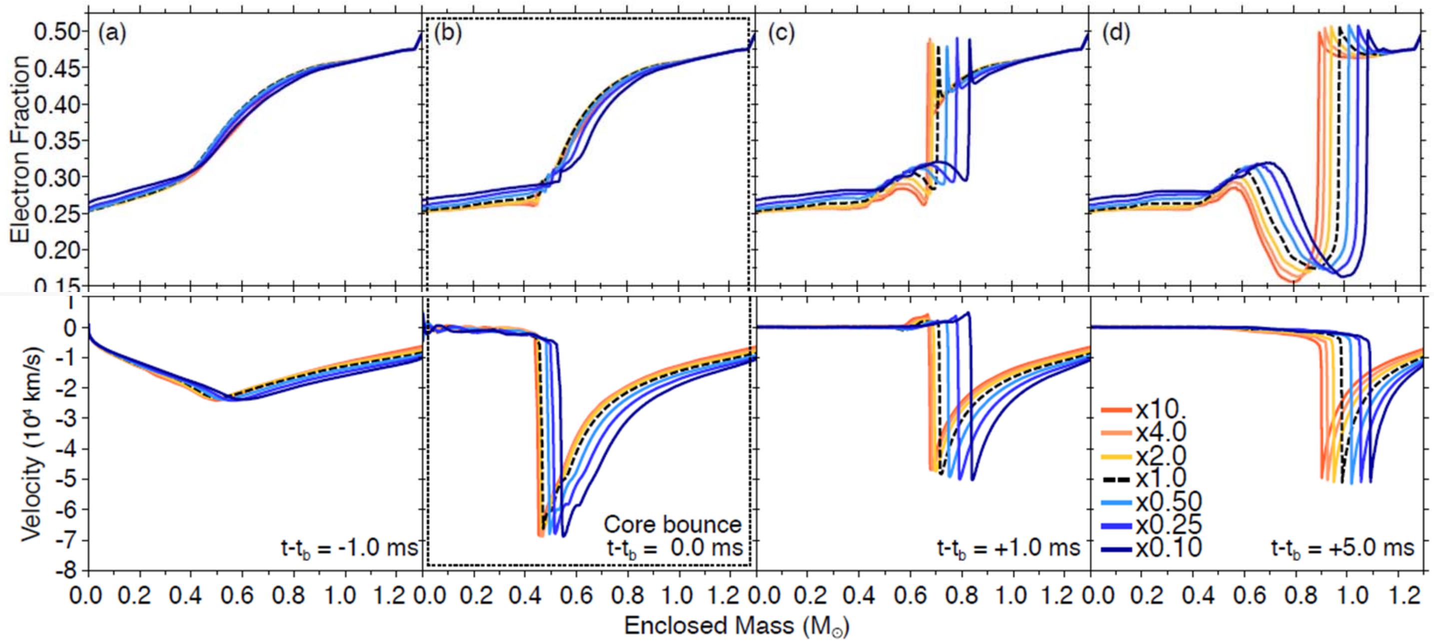

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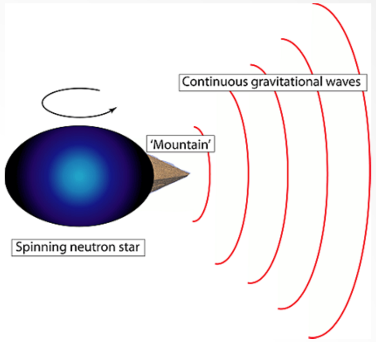
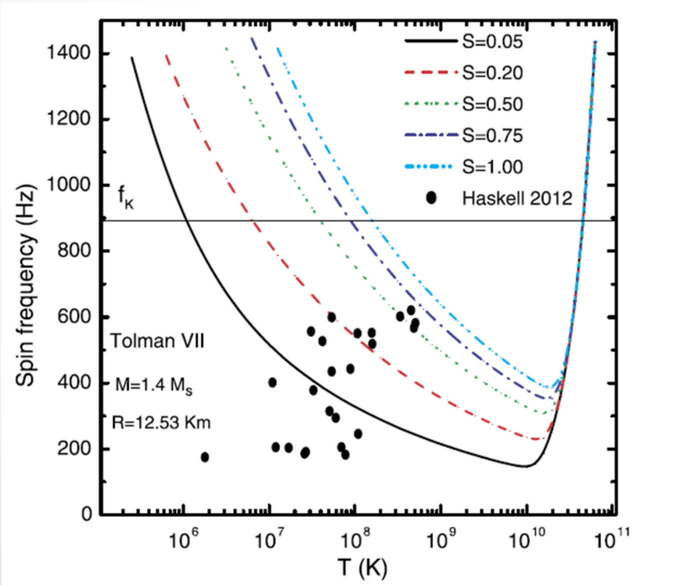
   f. **GW emission => EoS parameters**

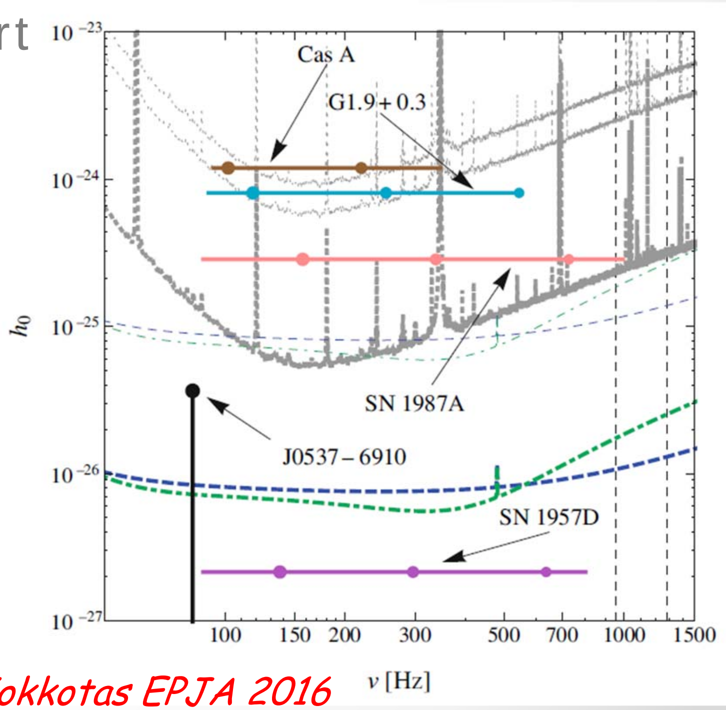
F- EoS and GW signals



- Spinning NS with asymmetric deformations (~1-10 Hz)
  - Elastic strains in the crust or magnetic fields in the core
  - Too weak for aLIGO and ET
- Unstable r-modes in young sources (~100-500 Hz)
  - Undamped by viscous dissipation if T and ν are high enough
  - Potentially detectable + EM counterpart
  - Very complex modelling



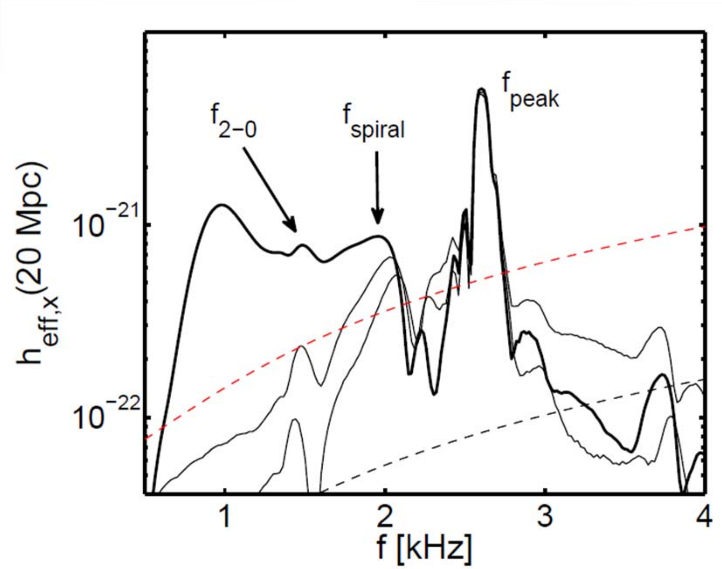
C.C.Moustakidis, Astrophys.Space Sci. 2016



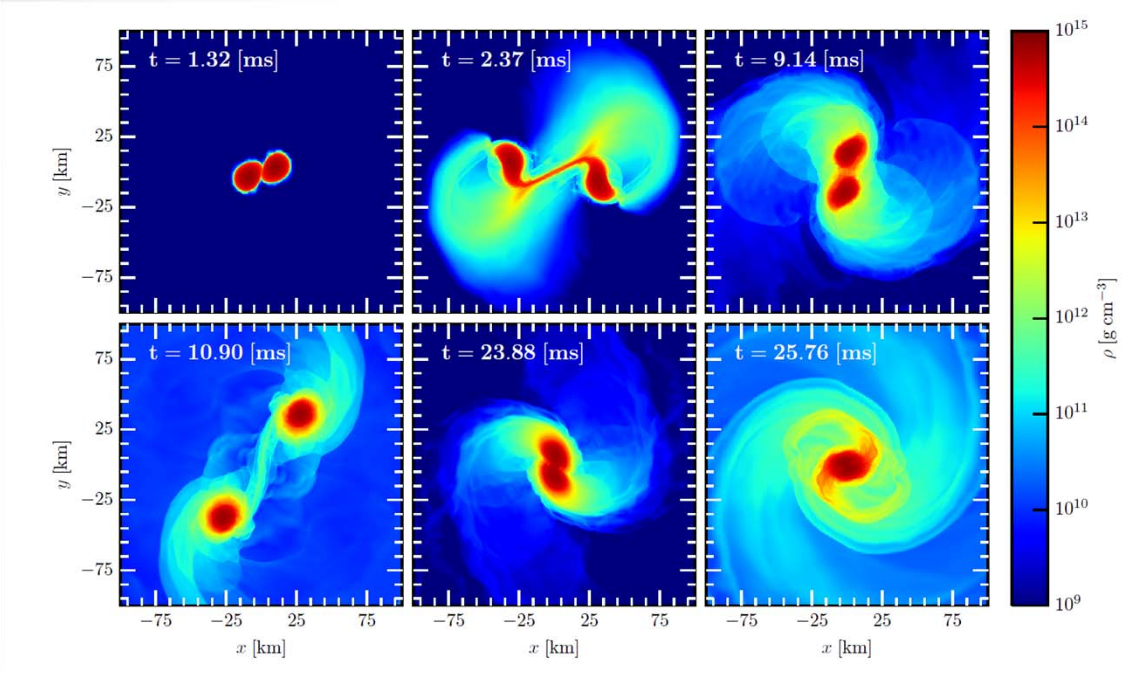
C.D.Kokkotas EPJA 2016

F- EoS and GW signals

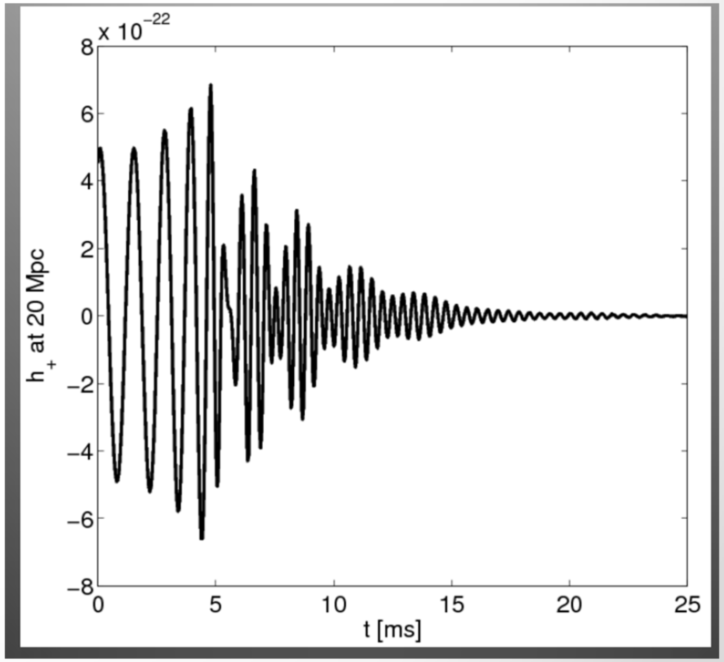
- NS mergers (~100-500 Hz)



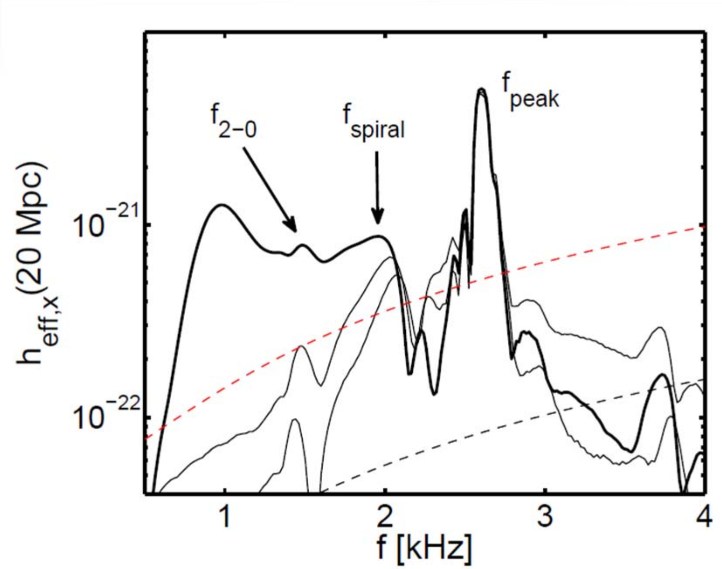
A.Bauswein, arXiV:1508.05493



A.Radice et al ArXiV 1601.02426



F- EoS and GW signals



- NS mergers (~100-500 Hz)
  - fundamental quadrupole fluid mode (f-peak) of the differentially rotating post-merger remnant
  - Detectable by aLIGO and ET (40/year)
  - Strongly correlated to the radius=>EoS
  - Robust signature

A.Bauswein, arXiV:1508.05493

F.Foucart et al, PRD93(2016)