LECTURE #1: NUCLEAR AND THERMONUCLEAR REACTIONS

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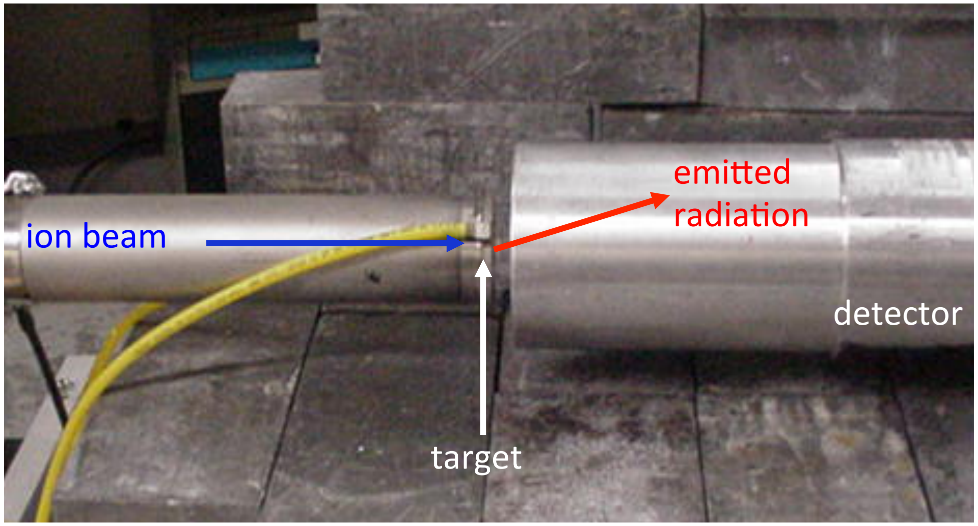
TUNL

NUCLEAR REACTIONS



Definition of cross section:

$$\sigma \equiv \frac{\left(\text{number of interactions per time}\right)}{\left(\text{number of incident particles per area per time}\right)\left(\text{number of target nuclei within the beam}\right)} = \frac{N_r}{N_0 N_t}$$

Unit: 1 barn=10^{-28} m^2

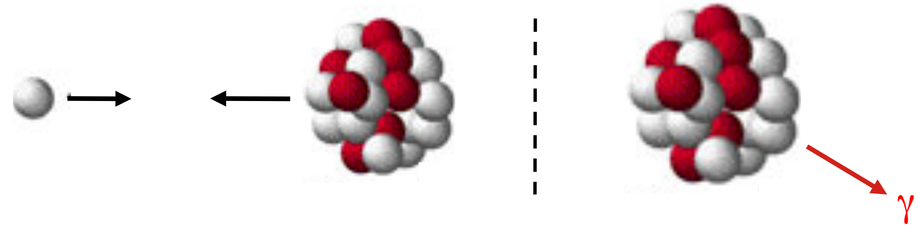
Example: p + p → d + e^+ + ν (first step of pp chain)

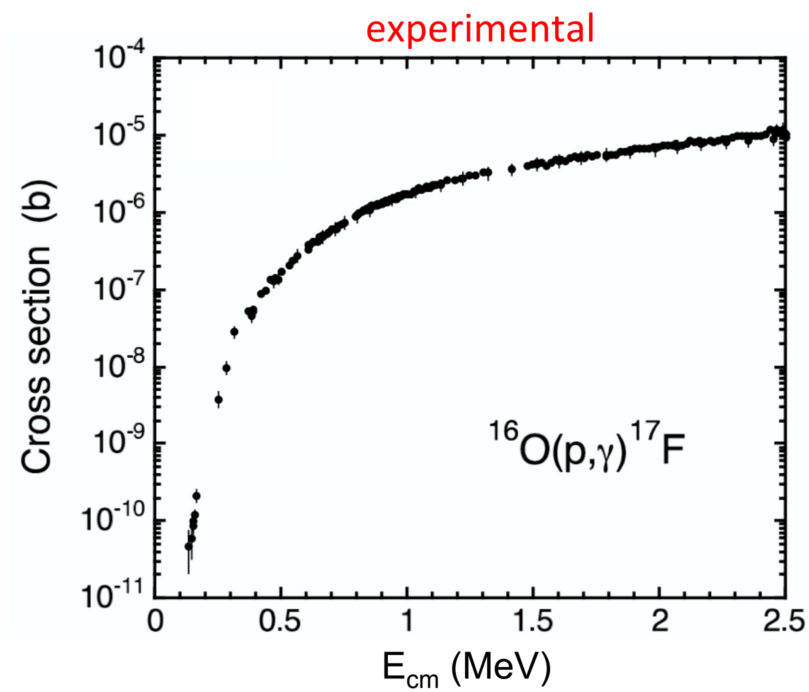
σ_{theo}=8x10^{-48} cm^2 at E_{lab}=1 MeV [E_{cm}=0.5 MeV]

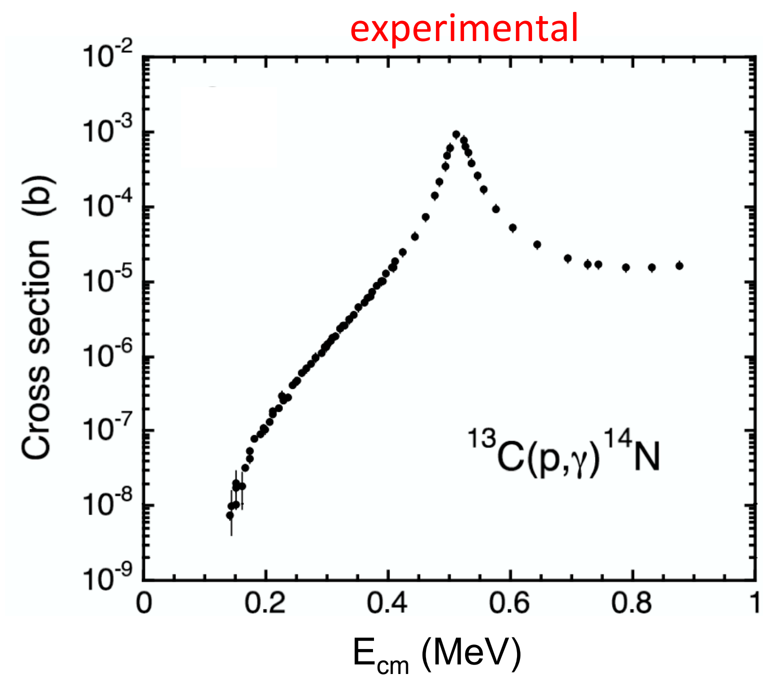
1 ampere (A) proton beam (6x10^{18} p/s) on dense proton target (10^{20} p/cm^2)

gives only 1 reaction in 6 years of measurement!

NUCLEAR CROSS SECTIONS







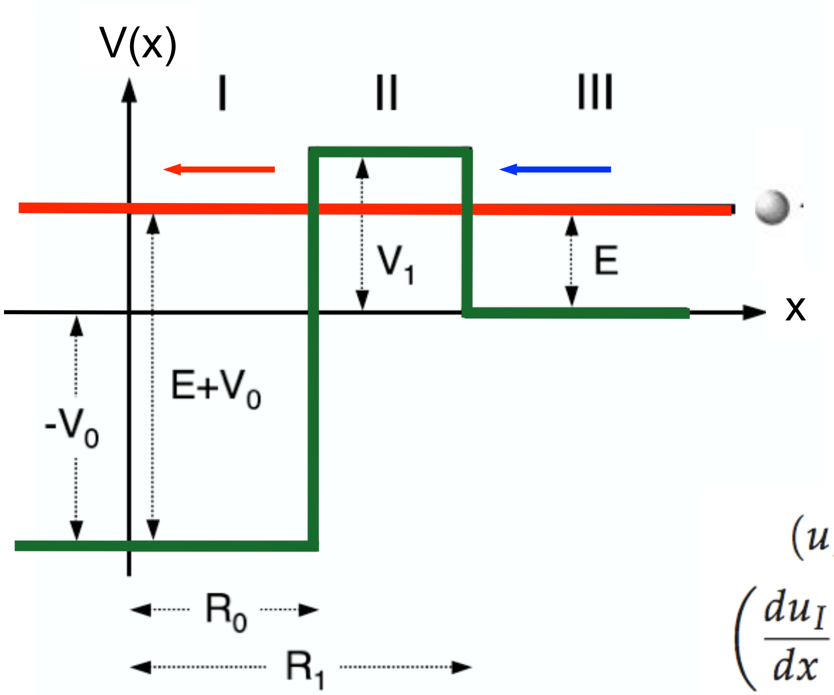
(i) why does the cross section fall drastically at low energies?

(ii) where is the peak in the cross section coming from?

SIMPLE 1D EXAMPLE

$$\frac{d^2u}{dr^2} + \hat{k}^2 u = 0$$

$$\boxed{\lambda = \frac{2\pi}{\hat{k}}}$$



wave function solutions:

$$u_I = Ae^{iKx} + Be^{-iKx} \qquad \hat{k}_I^2 = K^2 = \frac{2m}{\hbar^2}(E+V_0)$$

$$u_{II} = Ce^{-\kappa x} + De^{\kappa x} \qquad \hat{k}_{II}^2 = i^2\kappa^2 = i^2\frac{2m}{\hbar^2}(V_1 - E)$$

$$u_{III} = Fe^{ikx} + Ge^{-ikx} \qquad \hat{k}_{III}^2 = k^2 = \frac{2m}{\hbar^2}E$$

continuity condition:

$$(u_I)_{R_0} = (u_{II})_{R_0} \qquad (u_{II})_{R_1} = (u_{III})_{R_1}$$

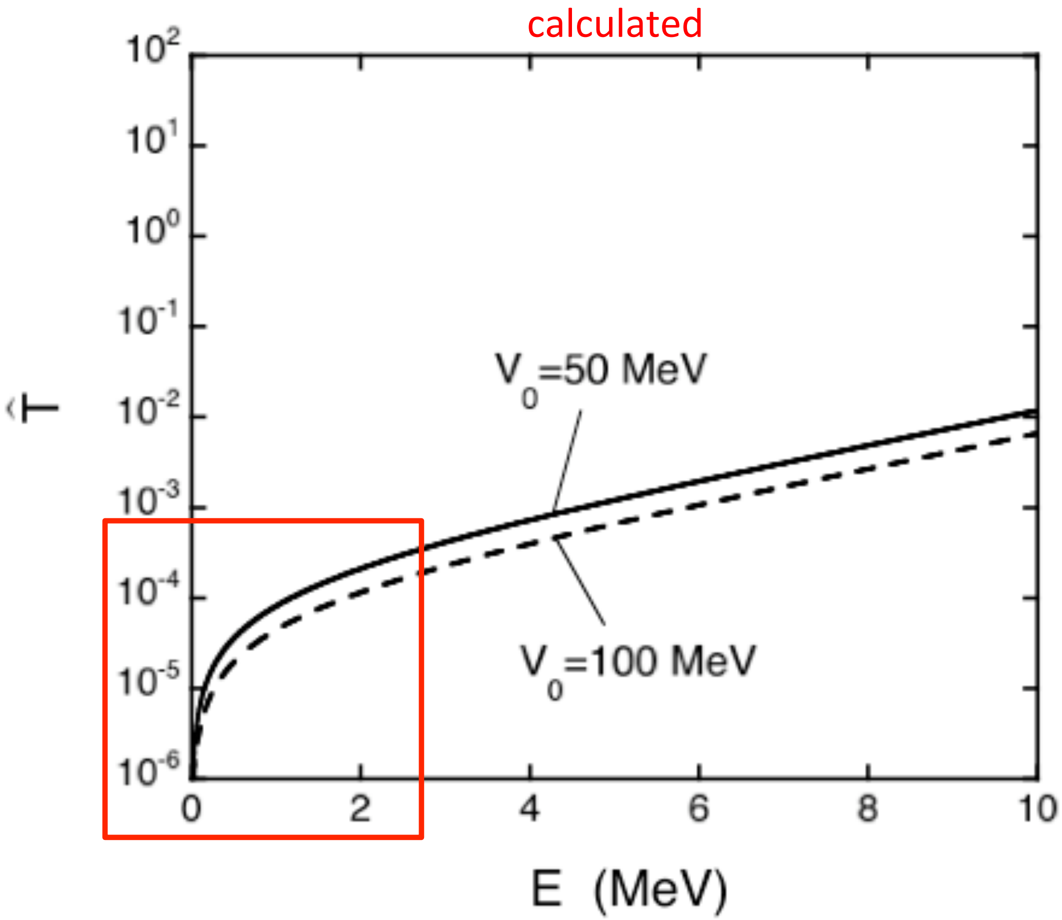
$$\left(\frac{du_I}{dx}\right)_{R_0} = \left(\frac{du_{II}}{dx}\right)_{R_0} \qquad \left(\frac{du_{II}}{dx}\right)_{R_1} = \left(\frac{du_{III}}{dx}\right)_{R_1}$$

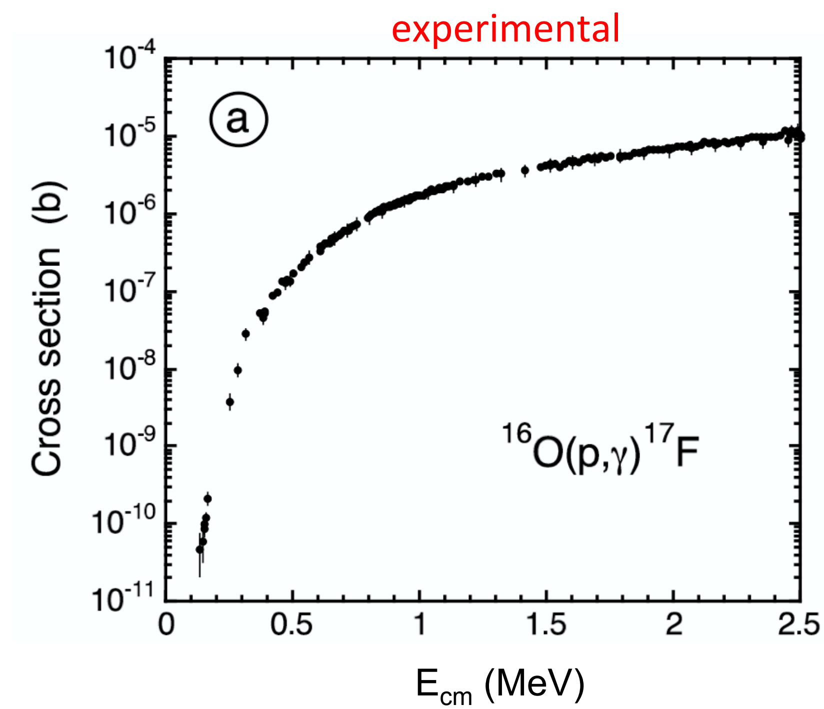
transmission coefficient: $$\hat{T} = \frac{K}{k}\frac{|B|^2}{|G|^2} \approx e^{-(2/\hbar)\sqrt{2m(V_1-E)}(R_1-R_0)}$$

(after lengthy algebra, and for the limit of low E)

"quantum tunneling"

OBSERVATIONS

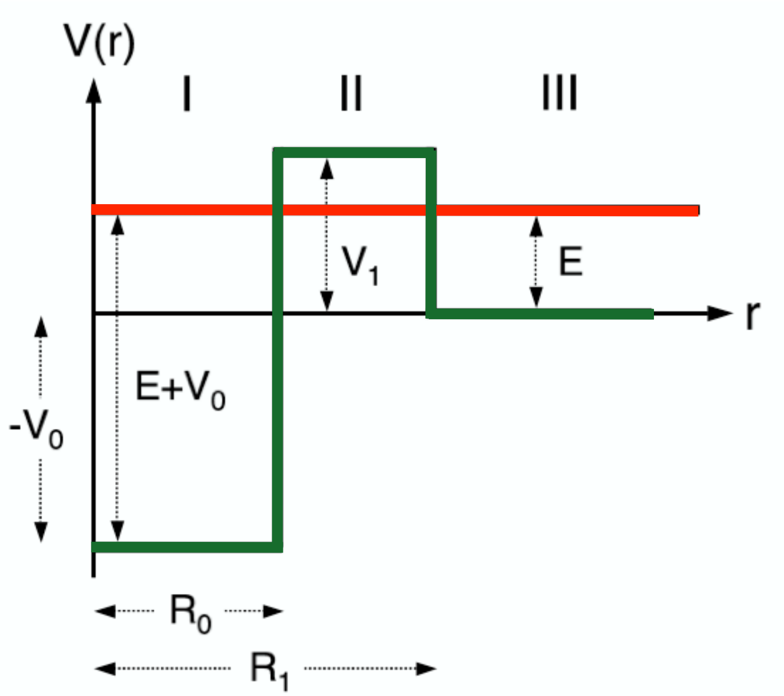




quantum tunneling is the reason for the strong drop in cross section at low energies!

BACK TO SIMPLE POTENTIAL, BUT NOW 3D

$$\lambda = \frac{2\pi}{\hat{k}}$$



wave function solutions:

$$u_I = A' \sin(Kr) \qquad \hat{k}_I^2 = K^2 = \frac{2m}{\hbar^2}(E + V_0)$$

$$u_{II} = Ce^{-\kappa r} + De^{\kappa r} \qquad \hat{k}_{II}^2 = i^2\kappa^2 = i^2\frac{2m}{\hbar^2}(V_1 - E)$$

$$u_{III} = F' \sin(kr + \delta_0) \qquad \hat{k}_{III}^2 = k^2 = \frac{2m}{\hbar^2}E$$

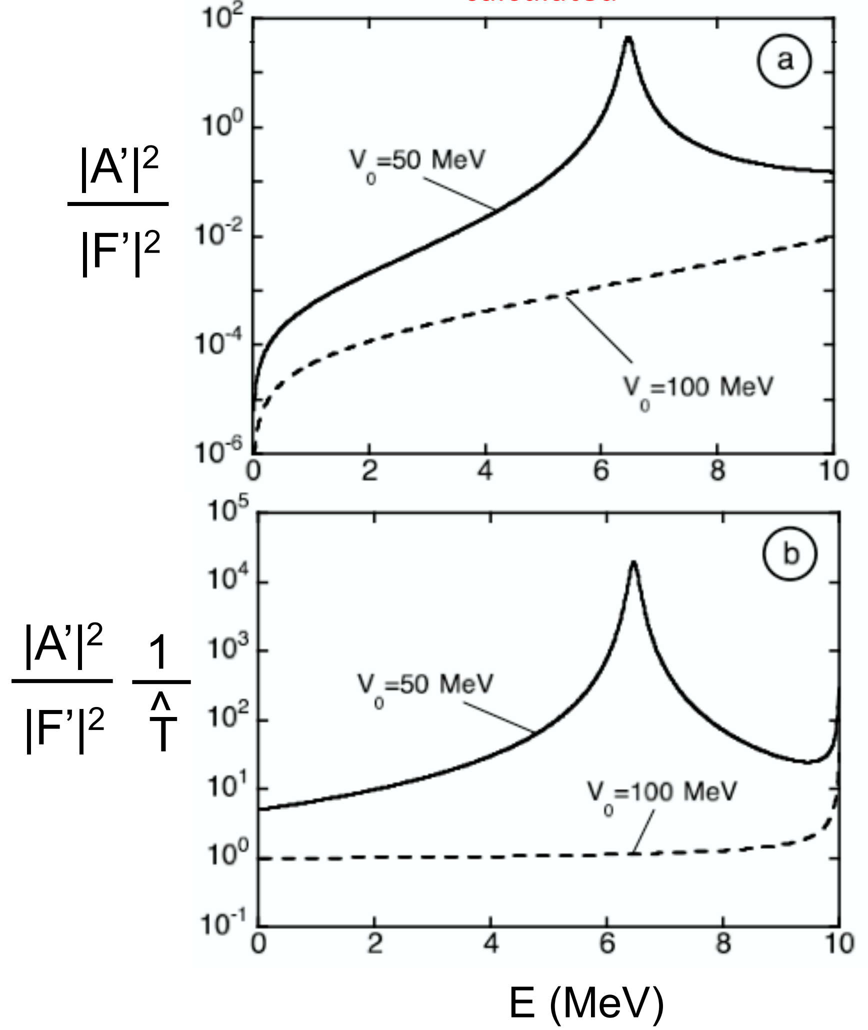
continuity condition…

wave intensity in interior region:
(after very tedious algebra)

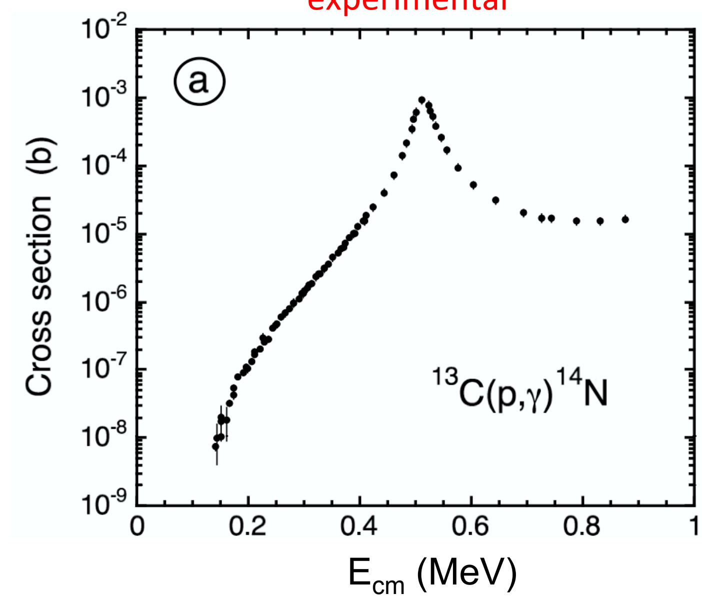
$$\frac{|A'|^2}{|F'|^2} = \left\{\sin^2(KR_0) + \left(\frac{K}{k}\right)^2 \cos^2(KR_0) + \sin^2(KR_0)\sinh^2(\kappa\Delta)\left[1 + \left(\frac{\kappa}{k}\right)^2\right] + \cos^2(KR_0)\sinh^2(\kappa\Delta)\left[\left(\frac{K}{\kappa}\right)^2 + \left(\frac{K}{k}\right)^2\right]\right.$$
$$\left. + \sin(KR_0)\cos(KR_0)\sinh(2\kappa\Delta)\left[\left(\frac{K}{\kappa}\right) + \left(\frac{K}{\kappa}\right)\left(\frac{\kappa}{k}\right)^2\right]\right\}^{-1}$$

COMPARISON TO OBSERVATION

calculated



experimental

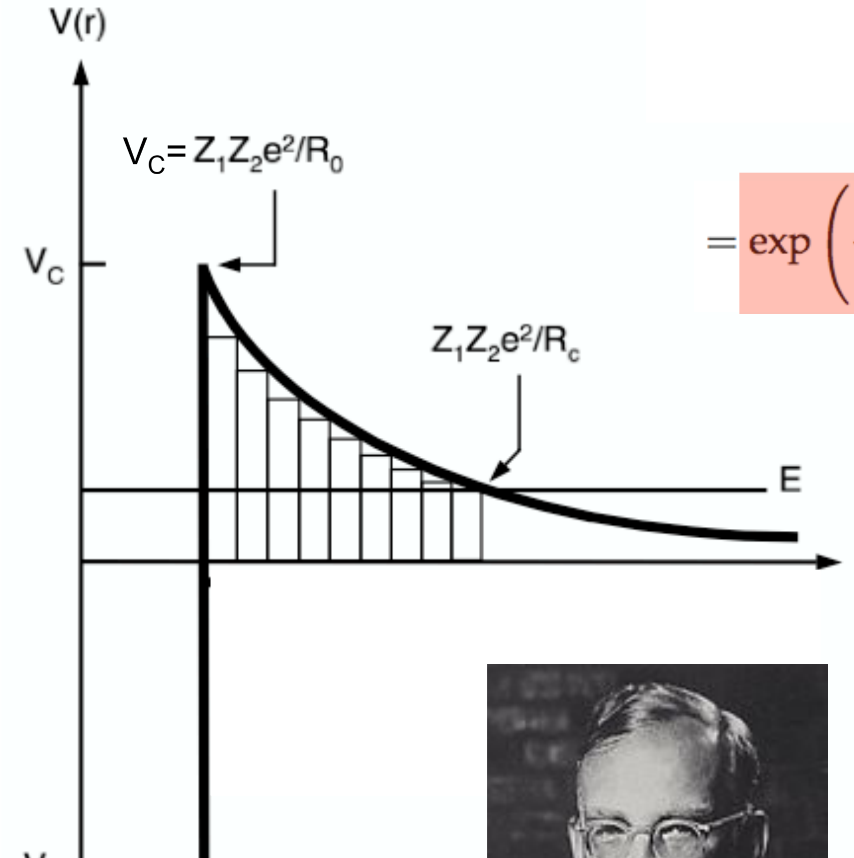


[change of potential depth V_0:
changes wavelength in interior region]

"resonance phenomenon"

a resonance results from favorable wave function matching conditions at the boundaries!

TRANSMISSION THROUGH COULOMB BARRIER



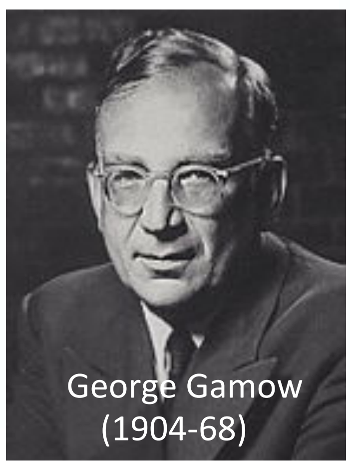
$$\hat{T} = \hat{T}_1 \cdot \hat{T}_2 \cdot \ldots \cdot \hat{T}_n \approx \exp\left[-\frac{2}{\hbar}\sum_i \sqrt{2m(V_i - E)}(R_{i+1} - R_i)\right]$$

$$\xrightarrow[n \text{ large}]{} \exp\left[-\frac{2}{\hbar}\int_{R_0}^{R_c} \sqrt{2m[V(r) - E]}\, dr\right]$$

$$= \exp\left(-\frac{2\pi}{\hbar}\sqrt{\frac{m}{2E}} Z_0 Z_1 e^2 \left[1 + \frac{2}{3\pi}\left(\frac{E}{V_C}\right)^{3/2}\right] + \frac{4}{\hbar}\sqrt{2mZ_0Z_1e^2R_0}\right)$$

[for low energies and zero angular momentum]

"Gamow factor" $e^{-2\pi\eta}$


George Gamow (1904-68)

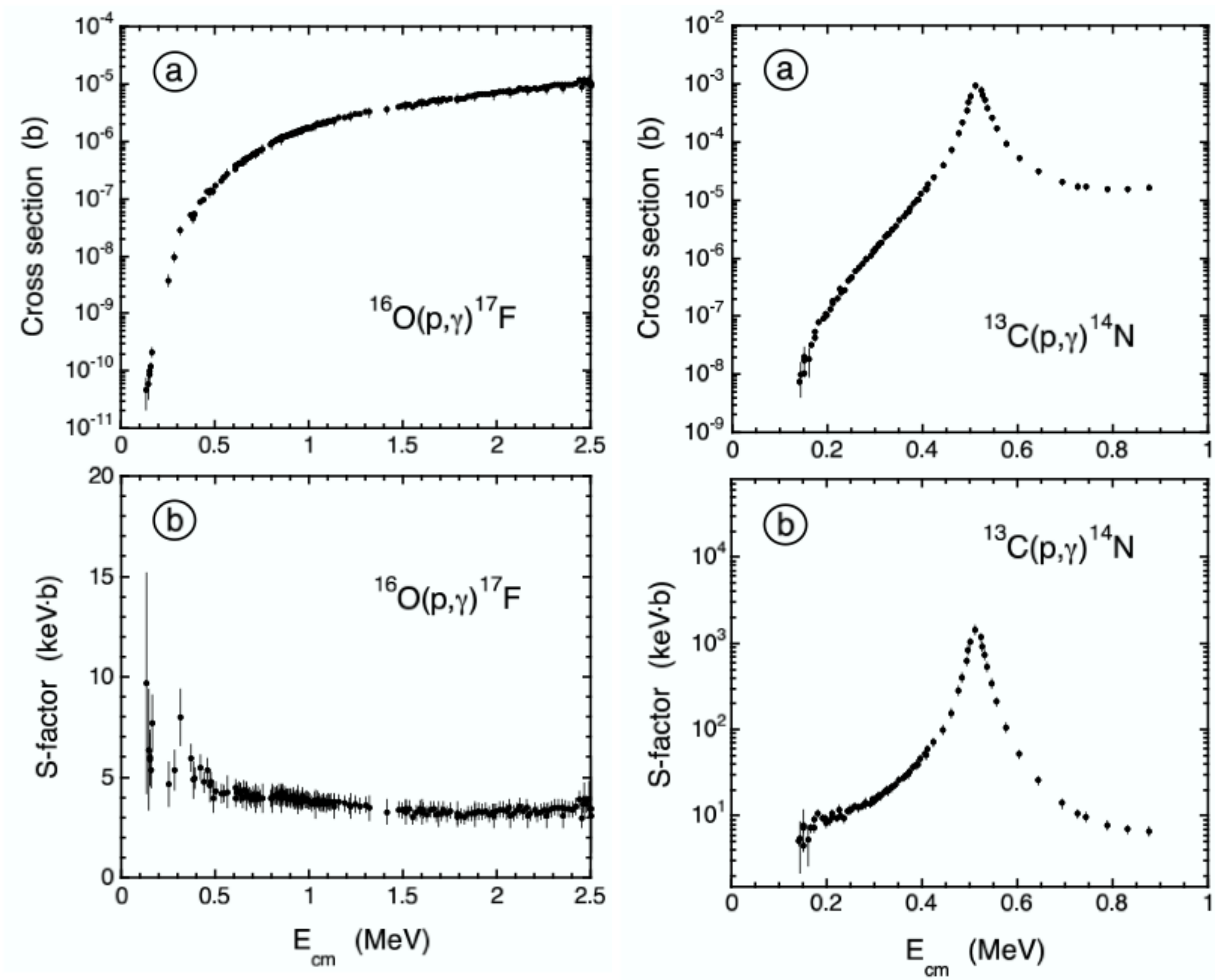
$$\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} S(E)$$

"astrophysical S-factor"

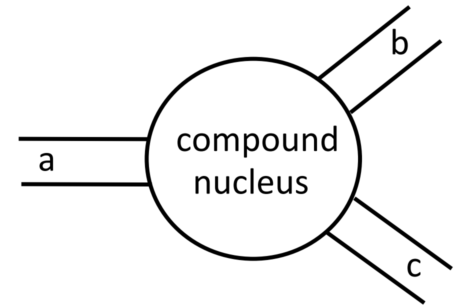
COMPARISON: S-FACTORS AND CROSS SECTIONS

cross sections

S-factors



THEORY: BREIT-WIGNER FORMULA



example: $^{22}Ne(\alpha,\gamma)^{26}Mg$
$^{22}Ne(\alpha,n)^{25}Mg$

Eugene Wigner
(1902-95)
Nobel Prize 1963

de Broglie wavelength

partial widths for incoming and outgoing channel

$$\sigma_{BW}(E) = \frac{\lambda^2}{4\pi} \underbrace{\frac{(2J+1)(1+\delta_{01})}{(2j_0+1)(2j_1+1)}}_{\text{spin factor}} \frac{\Gamma_a \Gamma_b}{(E_r - E)^2 + \Gamma^2/4}$$

resonance energy

total width

used for:
- for fitting data to deduce resonance properties
- for “narrow-resonance” thermonuclear reaction rates
- for extrapolating cross sections when no measurements exist
- for experimental yields when resonance cannot be resolved

THERMONUCLEAR REACTIONS

for a stellar plasma: kinetic energy for reaction derives from thermal motion:

"thermonuclear reaction"

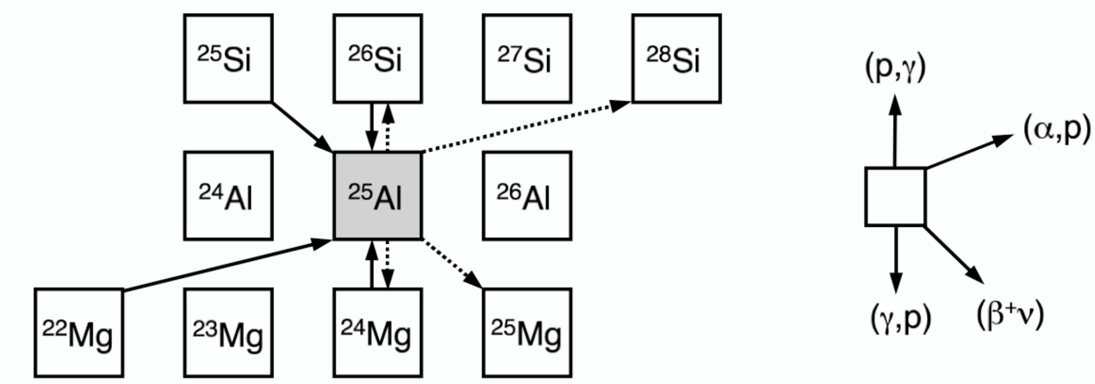
for a reaction 0 + 1 → 2 + 3 we find from the definition of σ (see earlier) a "reaction rate":

$$r_{01} = N_0 N_1 \int_0^\infty v P(v) \sigma(v)\, dv \equiv N_0 N_1 \langle \sigma v \rangle_{01}$$

for a Maxwell-Boltzmann distribution:

$$\boxed{\langle \sigma v \rangle_{01} = \left(\frac{8}{\pi m_{01}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E)\, e^{-E/kT}\, dE}$$

INTERPLAY OF REACTIONS IN PLASMA

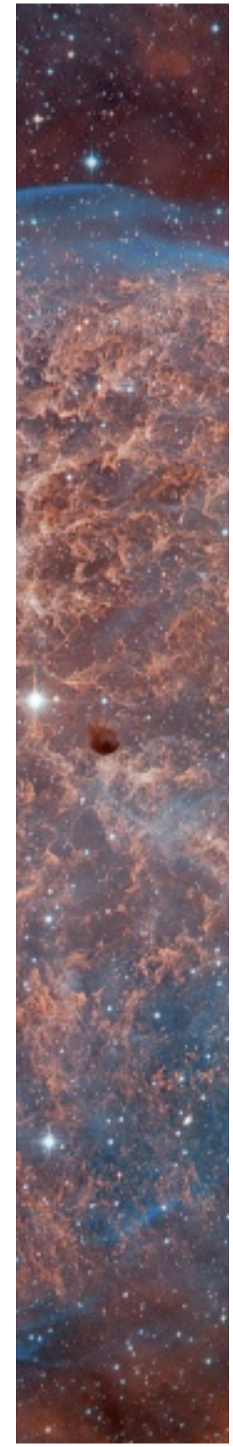


$$
\begin{aligned}
\frac{d(N_{^{25}\mathrm{Al}})}{dt} = & \left. N_{\mathrm{H}} N_{^{24}\mathrm{Mg}} \langle \sigma v \rangle_{^{24}\mathrm{Mg}(\mathrm{p},\gamma)} + N_{^{4}\mathrm{He}} N_{^{22}\mathrm{Mg}} \langle \sigma v \rangle_{^{22}\mathrm{Mg}(\alpha,\mathrm{p})} \right. \\
& \left. + N_{^{25}\mathrm{Si}} \lambda_{^{25}\mathrm{Si}(\beta^{+}\nu)} + N_{^{26}\mathrm{Si}} \lambda_{^{26}\mathrm{Si}(\gamma,\mathrm{p})} + \ldots \right\} \text{production} \\
& \left. - N_{\mathrm{H}} N_{^{25}\mathrm{Al}} \langle \sigma v \rangle_{^{25}\mathrm{Al}(\mathrm{p},\gamma)} - N_{^{4}\mathrm{He}} N_{^{25}\mathrm{Al}} \langle \sigma v \rangle_{^{25}\mathrm{Al}(\alpha,\mathrm{p})} \right. \\
& \left. - N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\beta^{+}\nu)} - N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\gamma,\mathrm{p})} - \ldots \right\} \text{destruction}
\end{aligned}
$$

system of coupled differential equations: "nuclear reaction network"

solved numerically, see:

- Arnett, "Supernovae and Nucleosynthesis", Princeton University Press (1996)
- Longland et al., Astron. Astrophys. 563, A67 (2014)



SPECIAL CASE #1: NON-RESONANT S-FACTOR (smoothly varying)

$$\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} S(E)$$

$$N_A \langle \sigma v \rangle = \left(\frac{8}{\pi m_{01}} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E)\, e^{-E/kT}\, dE$$

$$= \left(\frac{8}{\pi m_{01}} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} S_0 \int_0^\infty e^{-2\pi\eta}\, e^{-E/kT}\, dE$$

"Gamow peak"

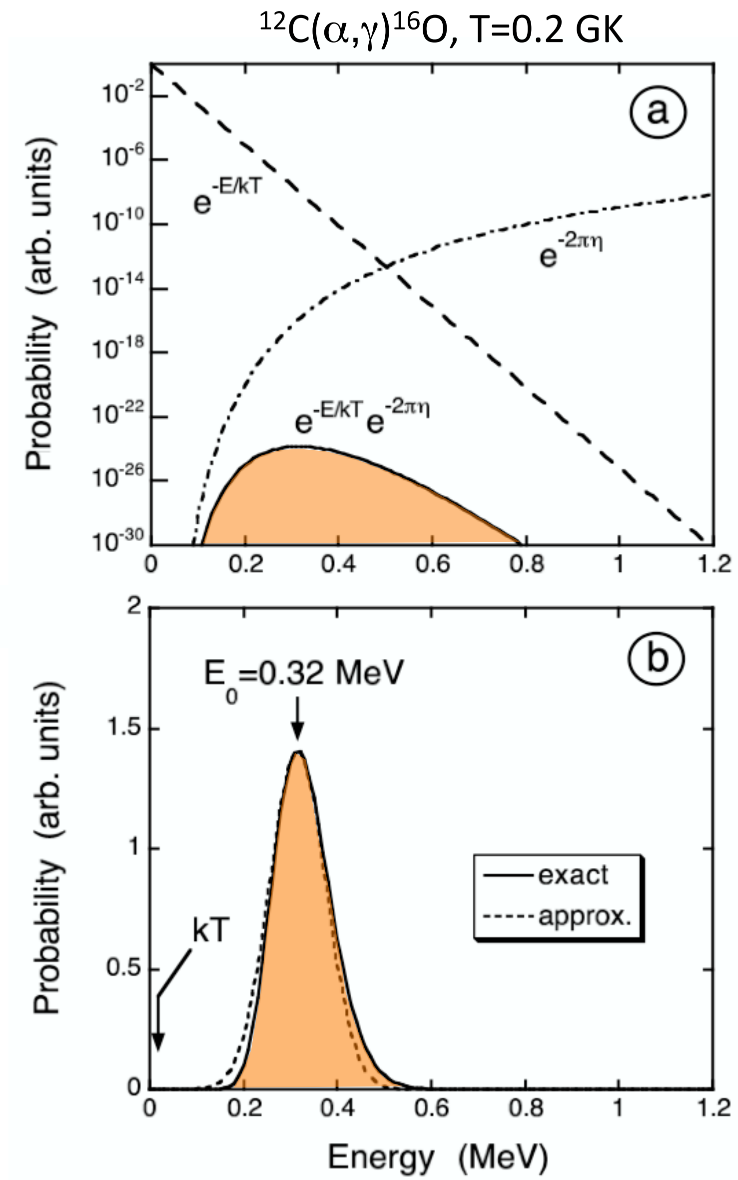
represents the energy range over which most nuclear reactions occur in a plasma!

location and 1/e width of Gamow peak:

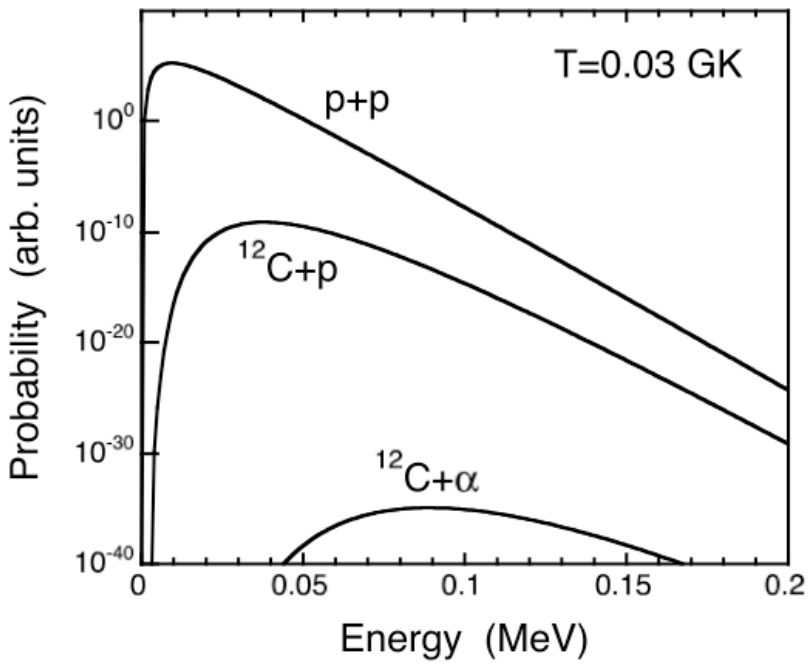
$$E_0 = \left[\left(\frac{\pi}{\hbar} \right)^2 \left(Z_0 Z_1 e^2 \right)^2 \left(\frac{m_{01}}{2} \right) (kT)^2 \right]^{1/3}$$

$$= 0.1220 \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0 + M_1} T_9^2 \right)^{1/3} \qquad \text{(MeV)}$$

$$\Delta = \frac{4}{\sqrt{3}} \sqrt{E_0 kT} = 0.2368 \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0 + M_1} T_9^5 \right)^{1/6} \qquad \text{(MeV)}$$



GAMOW PEAKS



$$e^{-2\pi\eta}\, e^{-E/kT}$$

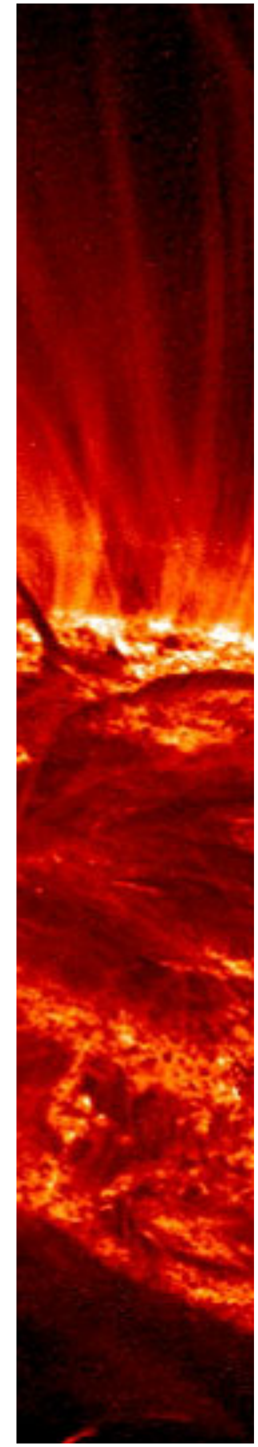
$$= \exp\left(-\frac{2\pi}{\hbar}\sqrt{\frac{m_{01}}{2E}}\, Z_0 Z_1 e^2 - \frac{E}{kT}\right)$$

important aspects:

(i) Gamow peak shifts to higher energy for increasing charges Z_p and Z_t

(ii) at same time, area under Gamow peak decreases drastically

conclusion: for a mixture of different nuclei in a plasma, those reactions with the smallest Coulomb barrier produce most of the energy and are consumed most rapidly [→ stellar burning stages]



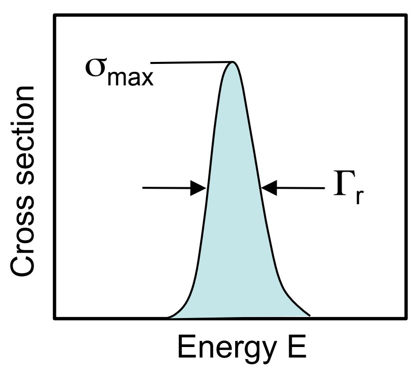
SPECIAL CASE #2: NARROW RESONANCES (Γ_i const over total Γ)

Breit-Wigner formula (energy-**independent** partial widths)

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

$$= N_A \frac{\sqrt{2\pi}\hbar^2}{(m_{01}kT)^{3/2}} e^{-E_r/kT} \omega \frac{\Gamma_a \Gamma_b}{\Gamma} 2\pi$$

- resonance energy needs to be known rather precisely
- takes into account only rate contribution at E_r



$$\omega\gamma \propto \sigma_{max} \cdot \Gamma_r$$

"resonance strength" $\omega\gamma$:
- proportional to area under narrow resonance curve
- energy-dependence of σ not important

SPECIAL CASE #3: BROAD RESONANCES

Breit-Wigner formula (energy-**dependent** partial widths)

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

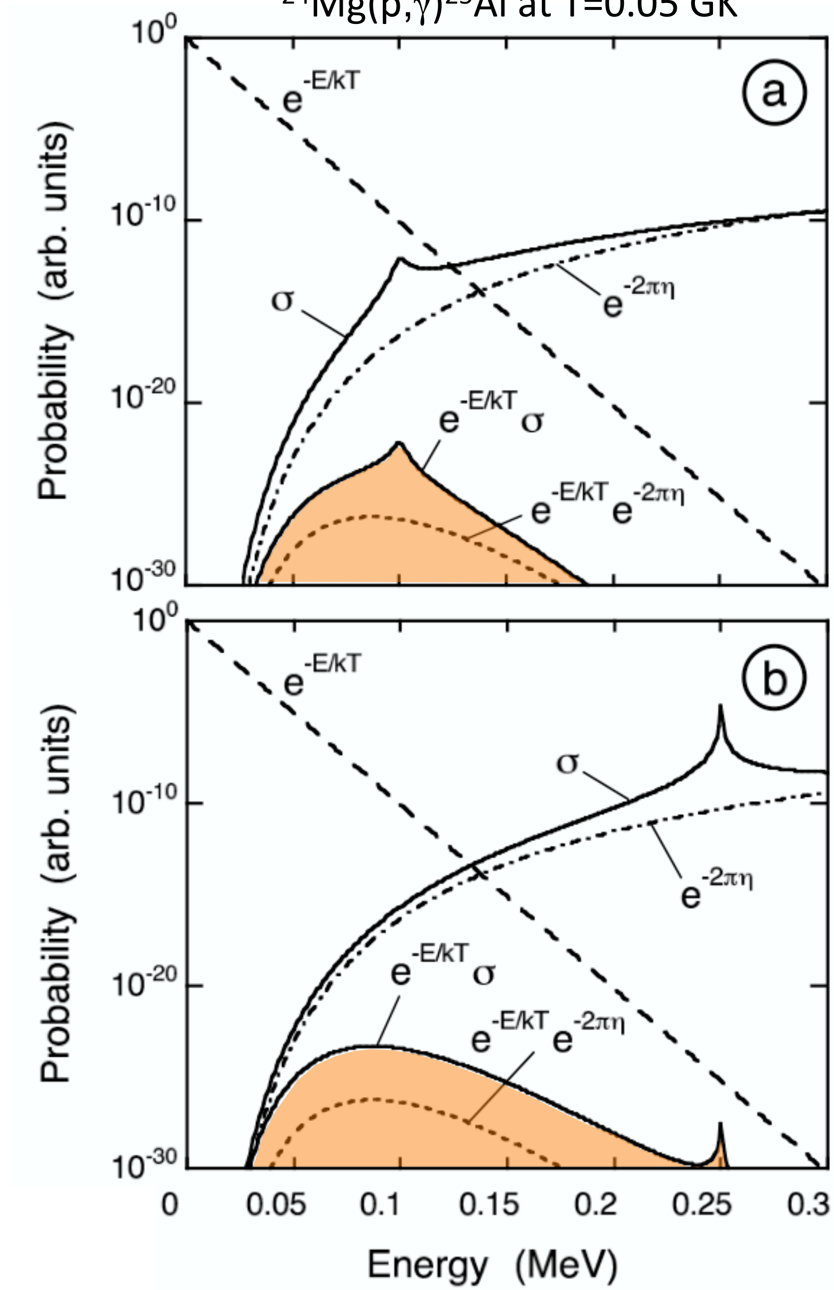
rate can be found from numerical integration

there are two contributions to the rate:

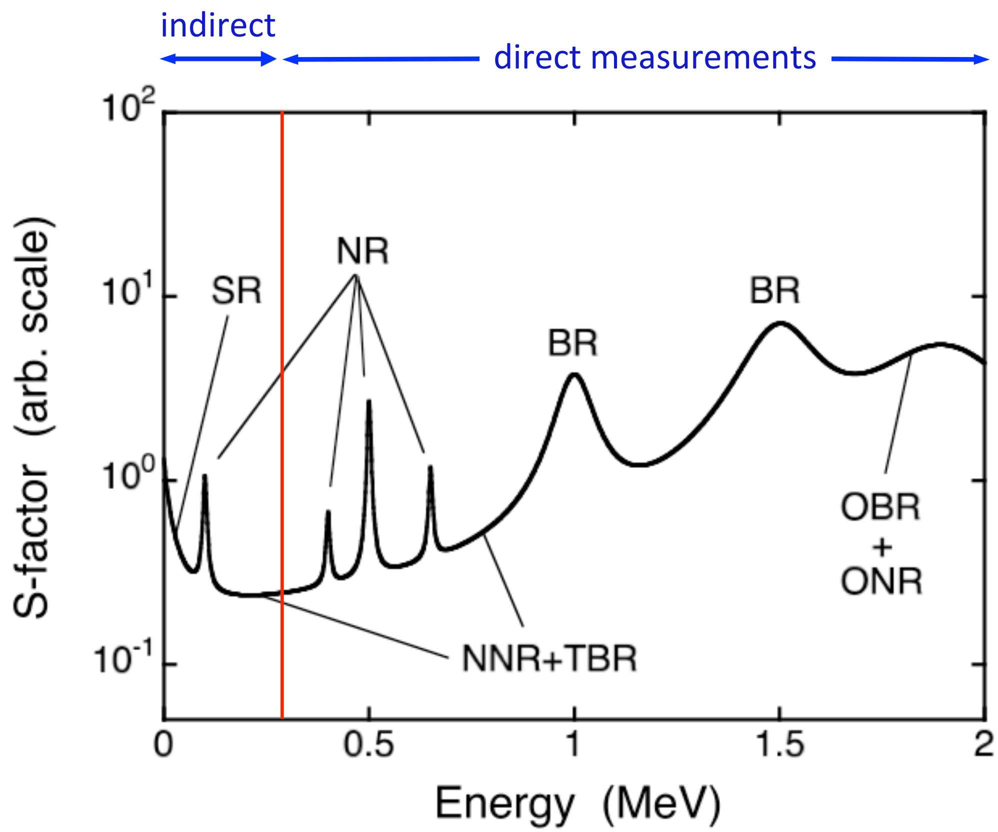
(i) from "narrow resonance" at E_r

(ii) from tail of broad resonance

$^{24}Mg(p,\gamma)^{25}Al$ at T=0.05 GK



TOTAL REACTION RATE



need to consider:

- non-resonant processes
- narrow resonances
- broad resonances
- subthreshold resonances
- interferences
- continuum

every nuclear reaction represents a special case !