Multi-wavelength observations of neutron stars to constrain their mass and radius

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Plan

1) The formation of neutron stars

2) Properties of neutron stars

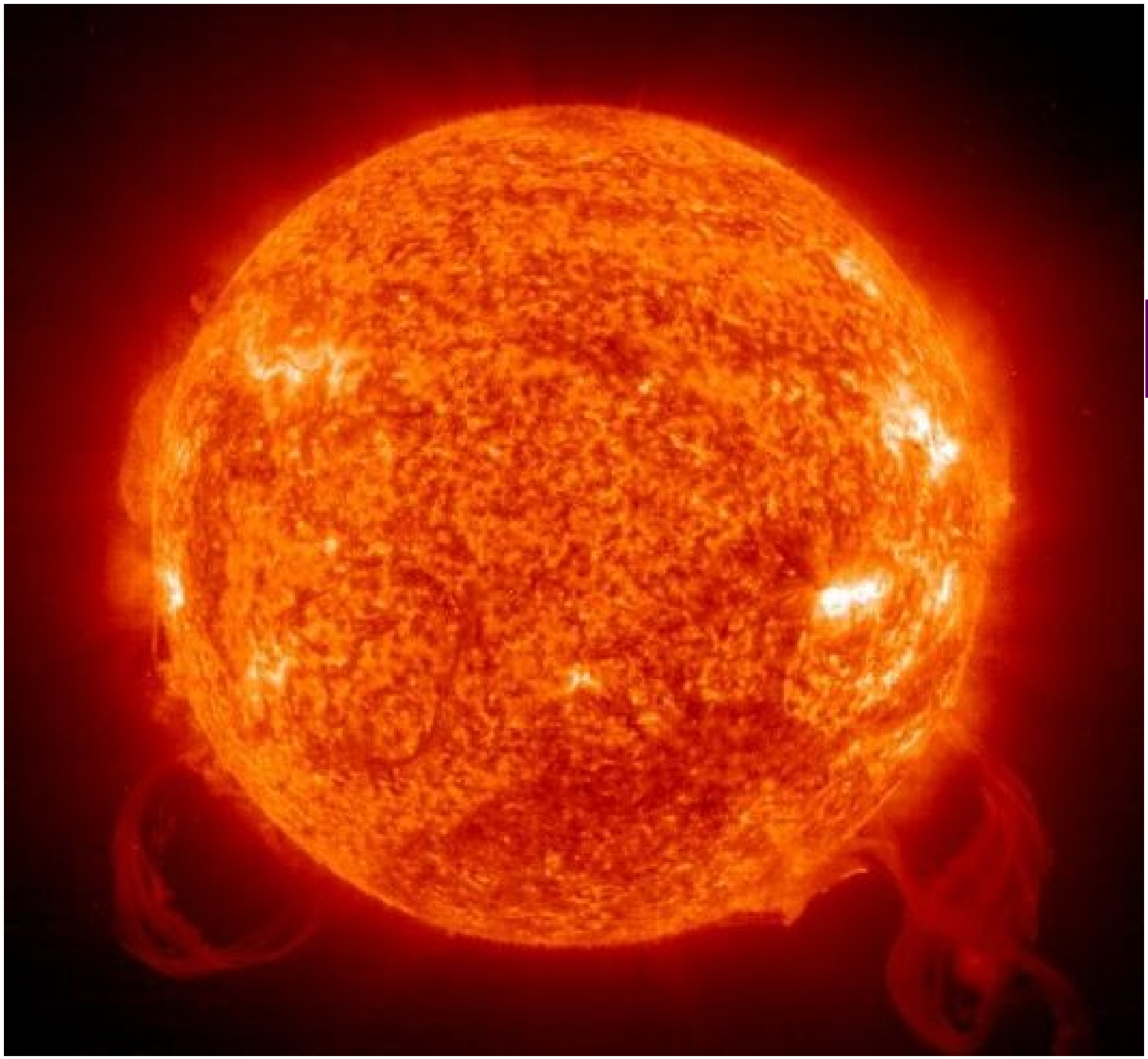
3) Neutron star interiors

4) Neutron star equation of state

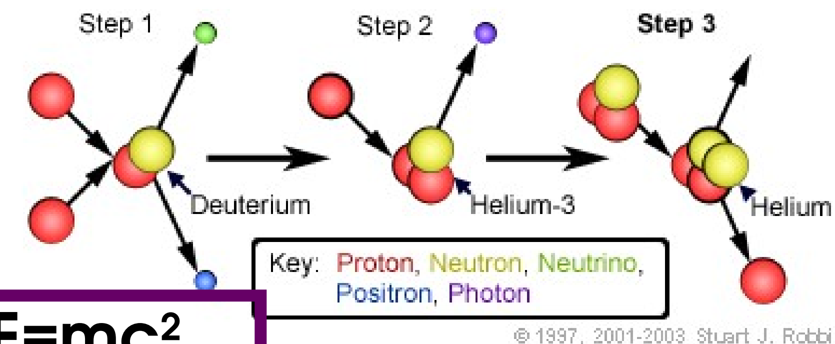
5) Determining the mass and radius of neutron stars

- Optical observations : Keplers laws
- Radio observations : Post-Keplerian parameters
- X-ray observations : Quasi periodic oscillations
- X-ray observations : Thermonuclear bursts
- X-ray observations : Thermal radiation
- Gravitational waves

Neutron star formation



$4\ {}^{1}H \rightarrow {}^{4}He + \text{energy}$

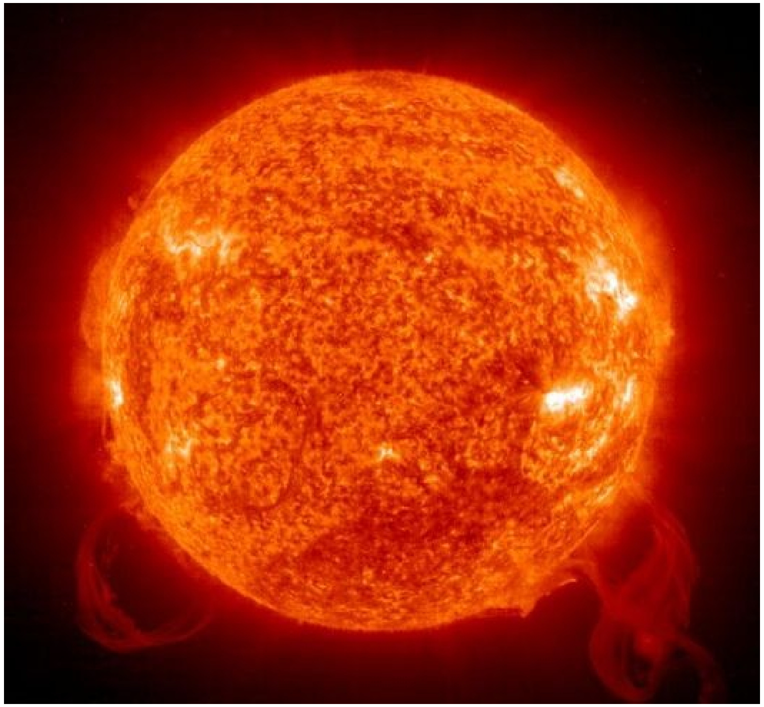


$\boxed{E=mc^2}$

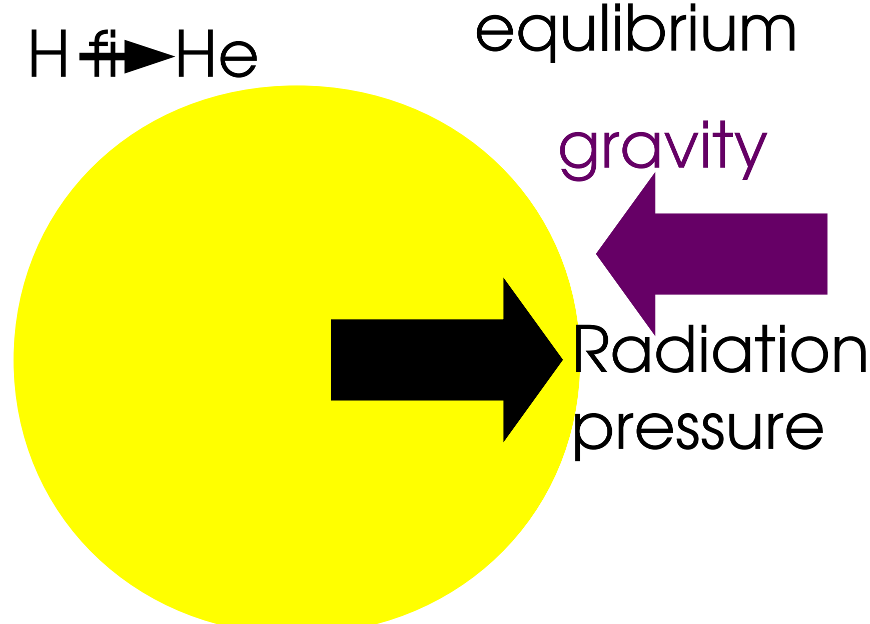
${}^{1}H = 1.67325\times10^{-27}$kg
${}^{4}He = 6.645\times10^{-27}$kg
Mass difference
$= 0.048\times10^{-27}$kg
$= 4.3\times10^{-12}$J

Required temperature for fusion :
~10^7 K for H→He
~10^8 K for He→C ...

Neutron star formation



Hydrostatic equlibrium



F = force
G = gravitational constant
M = mass
R = radius
A = area

For an ideal gas,

$$PV = kNT$$

and

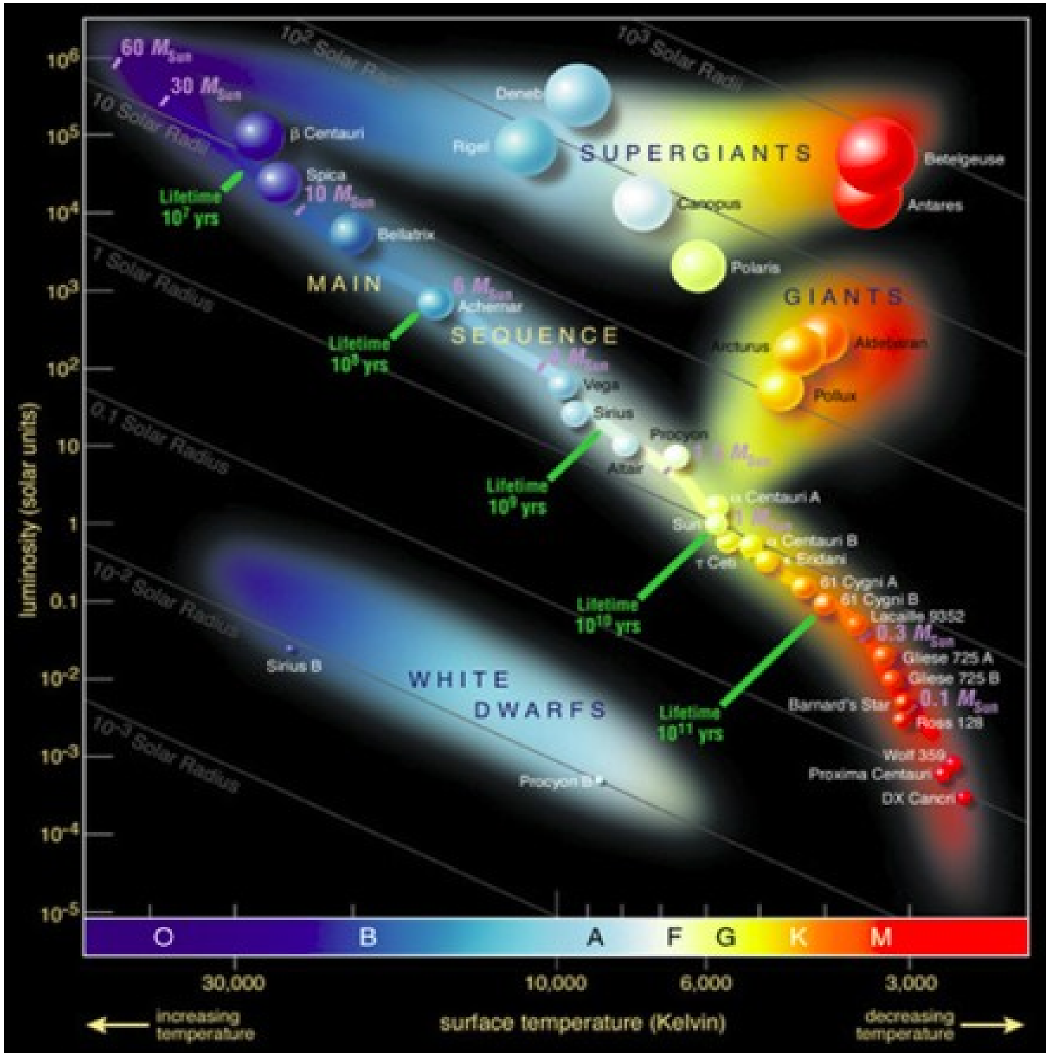
$$F = \frac{G M_1 M_2}{R^2}$$

P = pressure
k = Boltzmann constant
N = number of particles
T = temperature

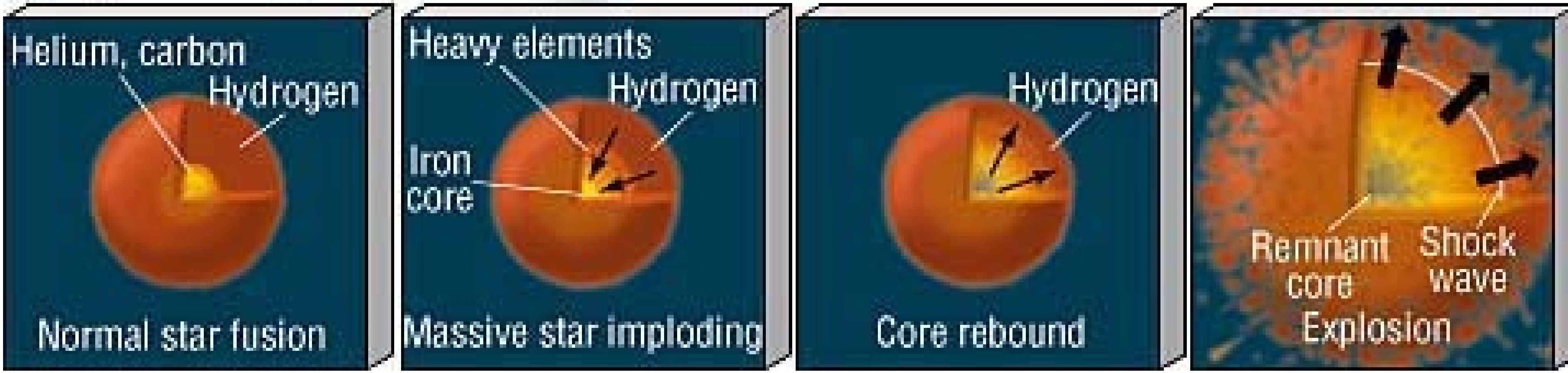
Neutron star formation

When ~10% of the core hydrogen has been used :

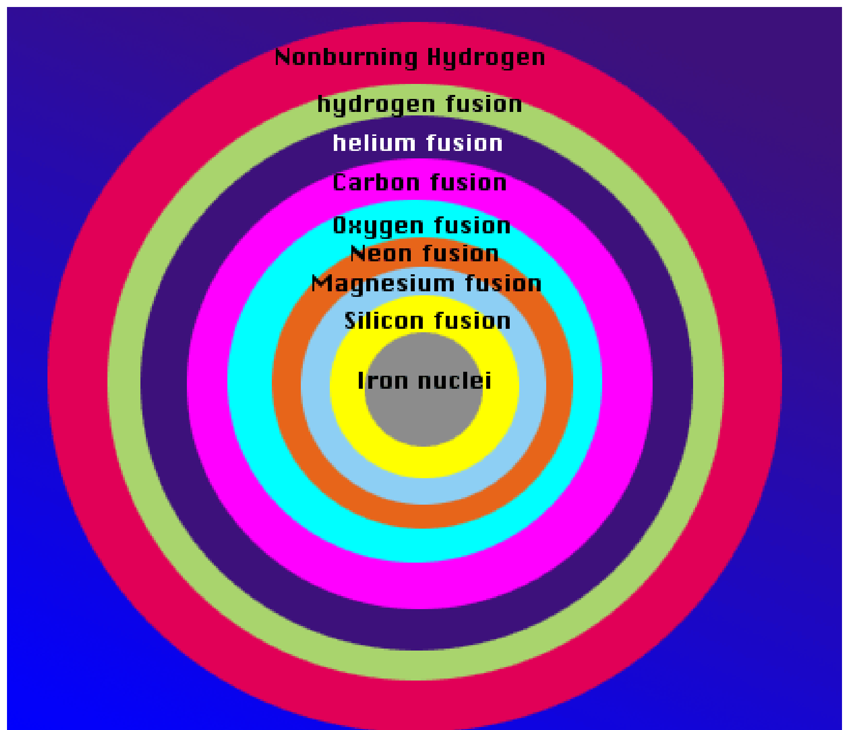
- the fusion stops
- force due to gravity exceeds force due to radiation pressure
- star collapses
- central temperature and pressure increase
- if temperature reaches ~10^8 K, helium can fuse
- balance is re-established
- star becomes a red giant



Neutron star formation : Stars of ~8-20M_{solar}

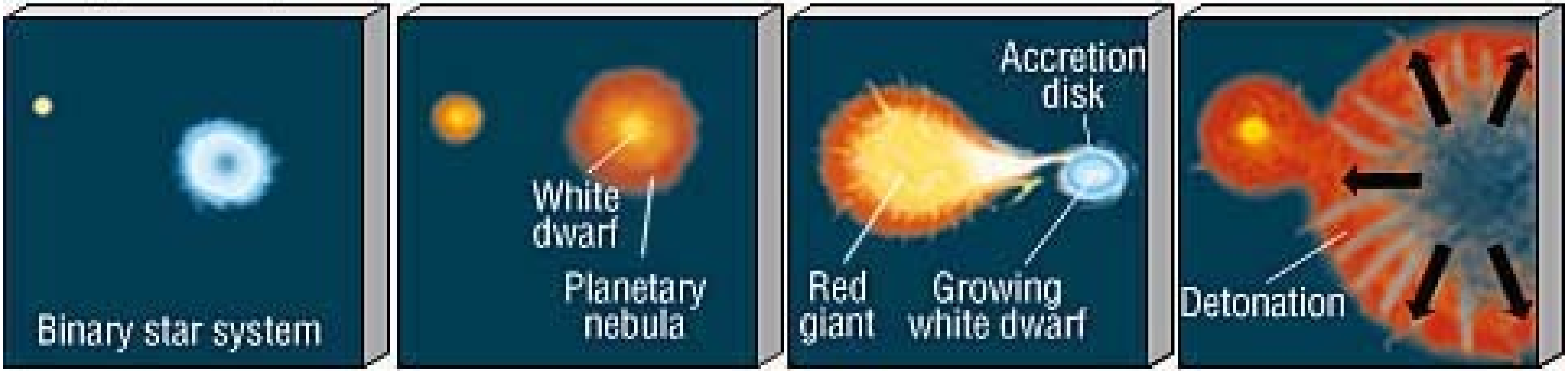


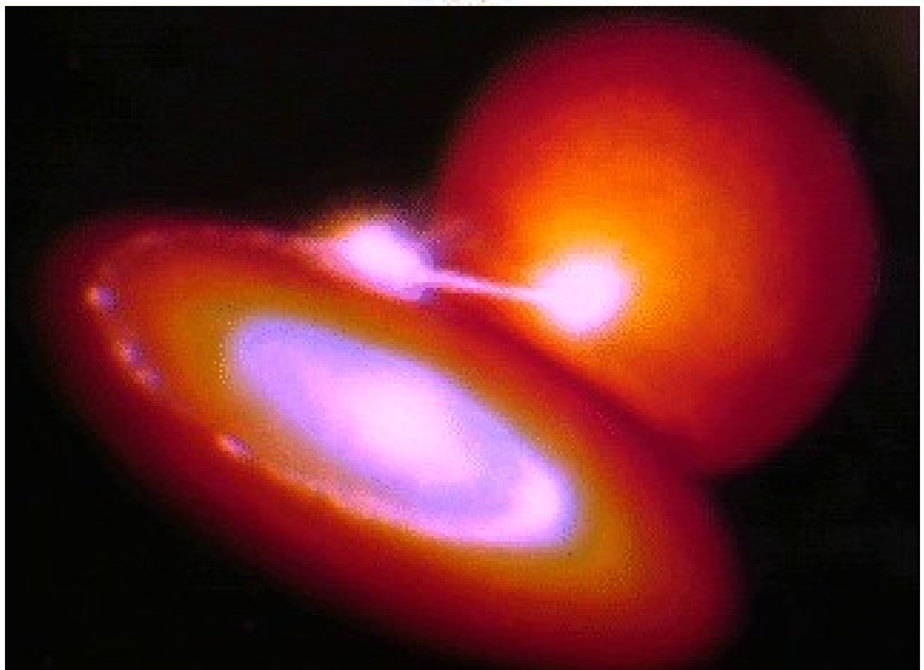
Supernovae type II



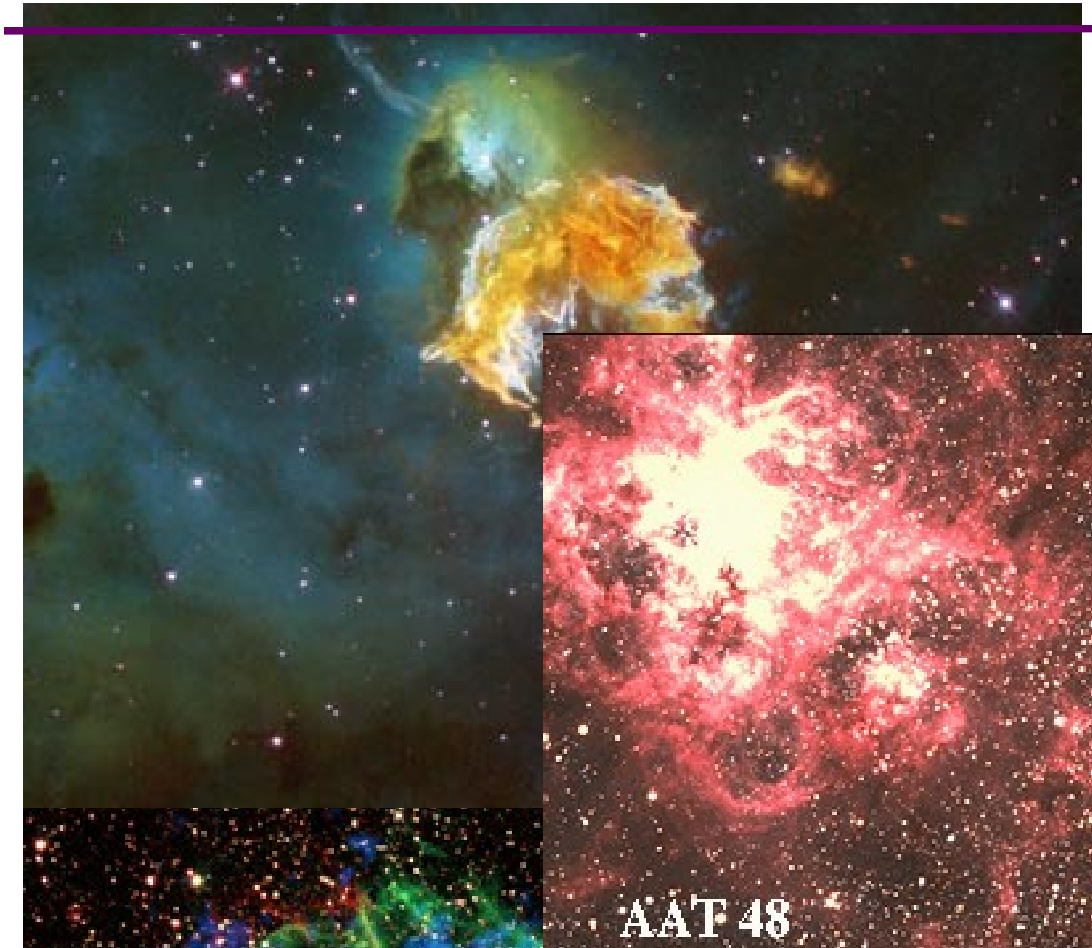
Neutron star formation

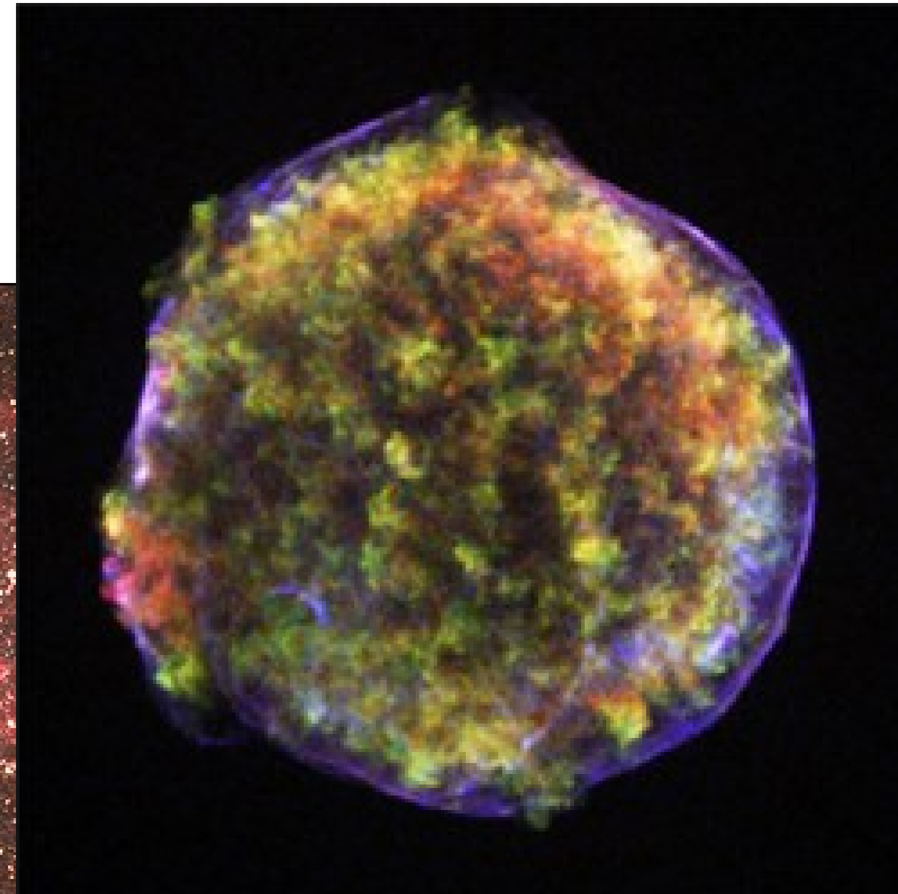
(a) Type- I Supernova

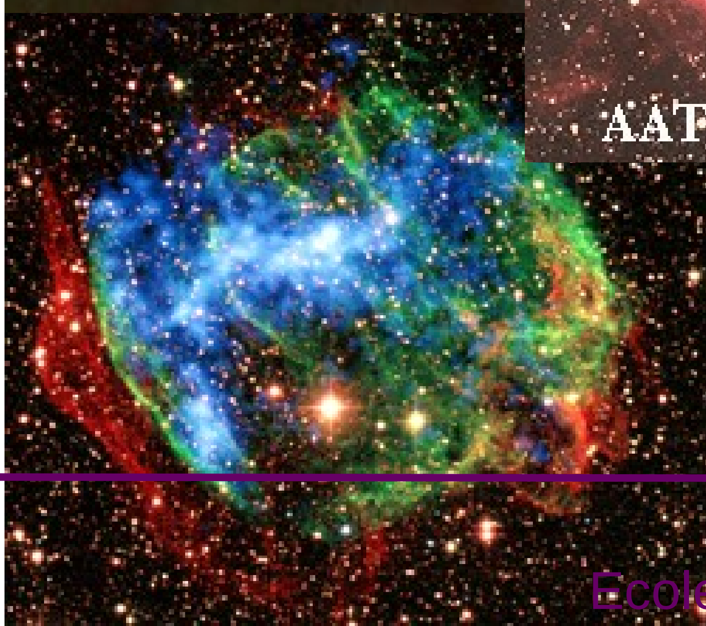




Neutron star formation





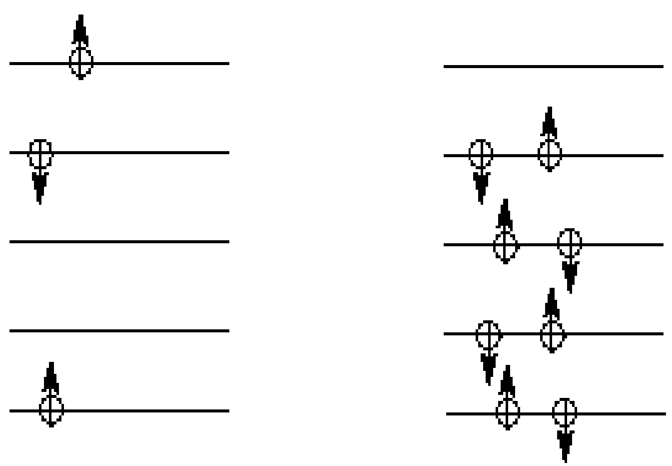


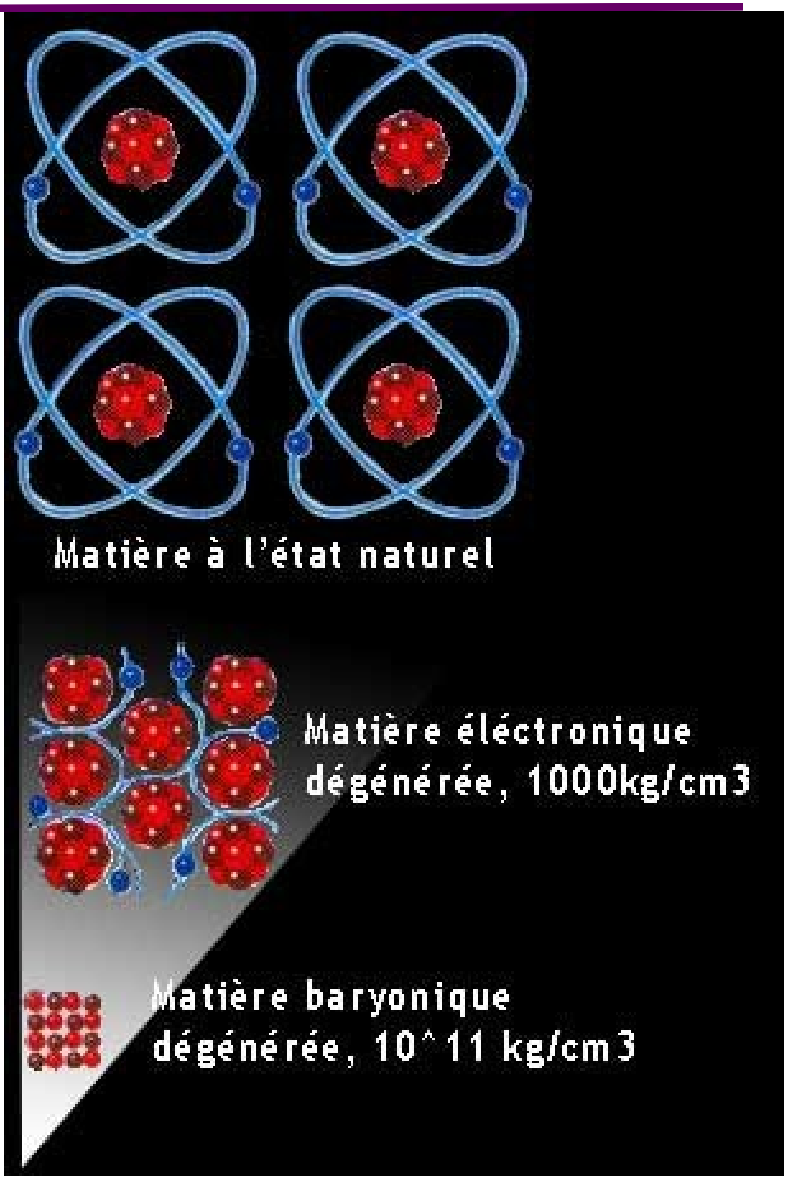
Supernova remnants

Neutron star formation

Because of the Pauli exclusion principal, which states that two electrons, or more generally two fermions, can not be in the same quantum state, the collapse of the nucleus is halted.

Successive shells of electrons are filled and the gas becomes degenerate





The electrons obey the Heisenberg uncertainty principal, so DΔp Δx ‡≥ħ

The momentum for each electron is then p~DΔp ~ħ/DΔx

Neutron star formation

If the electron density is n_e , the electron separation is $Dx \sim n_e^{-1/3}$, so the momentum for each electron is :

$$p \sim \hbar n_e^{1/3}$$

Rewriting pressure as a function of momentum (p), and using the fact that kinetic energy is :

$0.5mv^2 = 1.5$ NkT for an ideal gas

$\Rightarrow$ $0.333\ mv^2 = NkT$ and $P = NkT/V$ et $p=mv$
$\Rightarrow$ $0.333\ pv/V = P$ and $n_e \sim V^{-1}$
$\Rightarrow$ $0.333\ n_e pv = P$

where

$$P = \frac{1}{3} n_e p v = \frac{1}{3} n_e p \left(\frac{p}{m_e} \right)$$

Neutron star formation

So the pressure is given by :

$$P = \frac{1}{3} n_e p v = \frac{1}{3} n_e p \left(\frac{p}{m_e} \right)$$

Using $$p \sim \hbar n_e^{1/3}$$

$$\longrightarrow P = \frac{1}{3} n_e \left(\hbar n_e^{1/3} \right) \left(\frac{\hbar n_e^{1/3}}{m_e} \right) \sim n_e^{5/3} \sim \rho^{5/3}$$

So for a degenerate gas, $P \sim \rho^{5/3}$

Neutron star formation

If the electron speed approaches the speed of light : $P = \frac{1}{3} n_e \left(\hbar n_e^{1/3}\right) c \sim n_e^{4/3} \sim \rho^{4/3}$

The relativistic case is then :

$$P_c \simeq P_{e,rel}$$

$$\rho^2 R^2 \sim \rho^{4/3}$$

$$R^2 \sim \rho^{-2/3}$$

$$R^2 \sim \frac{M^{-2/3}}{R^{-2}}$$

$$1 \sim M^{-1/3}$$

Hydrostatic equilibrium :

$$\frac{dP}{dr} = \frac{-GM\rho}{r^2} = \frac{-G\,(4/3\,\pi\, r^3 \rho)\rho}{r^2} = -G\, 4/3\, \pi\, r\, \rho^2$$

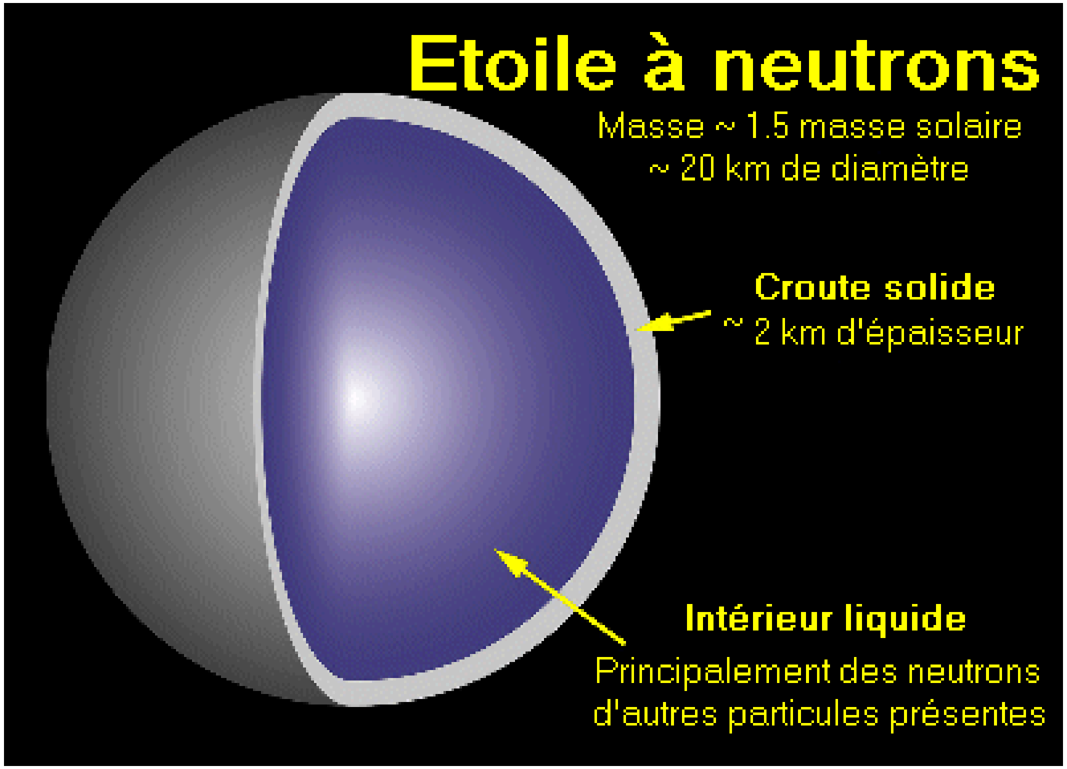
$$P_c = \int_0^R -4/3\, G\, \pi\, r\, \rho^2\, dr$$

$$= -2/3\, G\, \pi\, R^2 \rho^2$$

- The nucleus has a maximum mass of 1.4 M_{solar}
- This mass is called the *Chandrasekhar* mass after Subrahmanyan Chandrasekhar

Neutron star formation

- For a stellar core >1.4 M⊙, the matter must be degenerate.
- Electrons can not remain in their orbits and are forced into the atomic nucleus
- The electrons fuse with the protons to form confined neutrons.
- The Pauli exclusion principle makes it impossible to have 2 neutrons in the same quantum state in the same place.
- The force due to gravity Is opposed by the force due to the degeneracy pressure of the neutrons (baryonic degeneracy)



Neutron star properties

Neutron star
1.4 M_{solar}

Neutron star properties

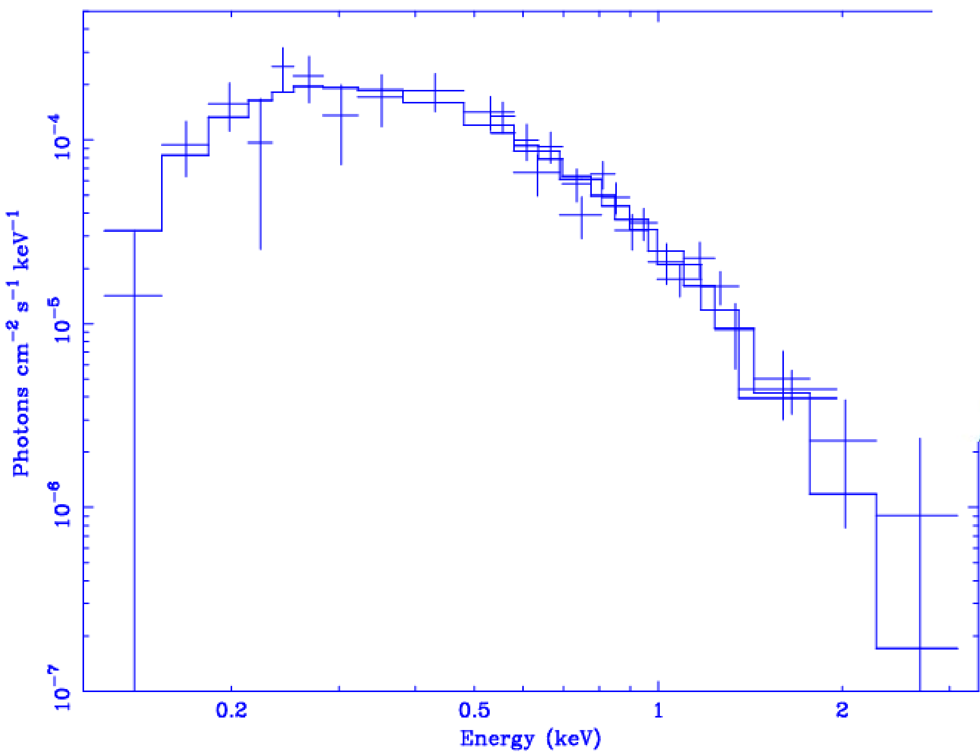
Temperature at birth:

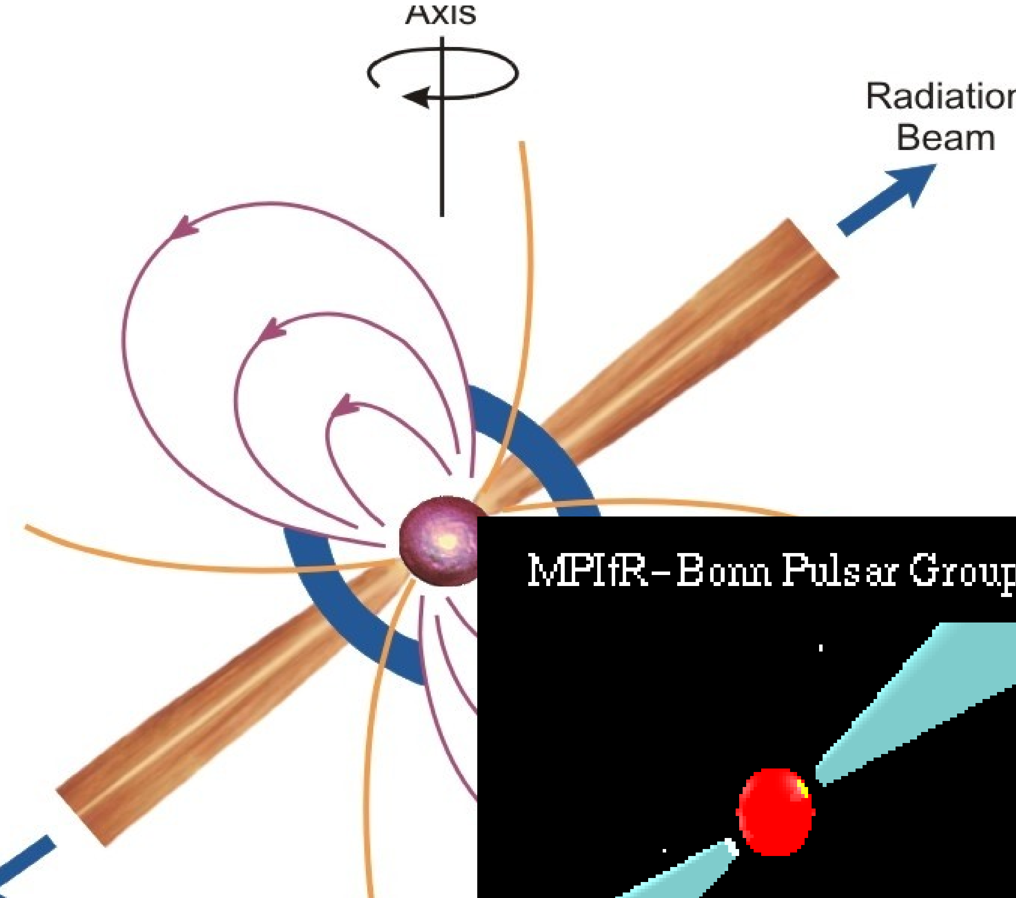
~ 1×10^{6} K

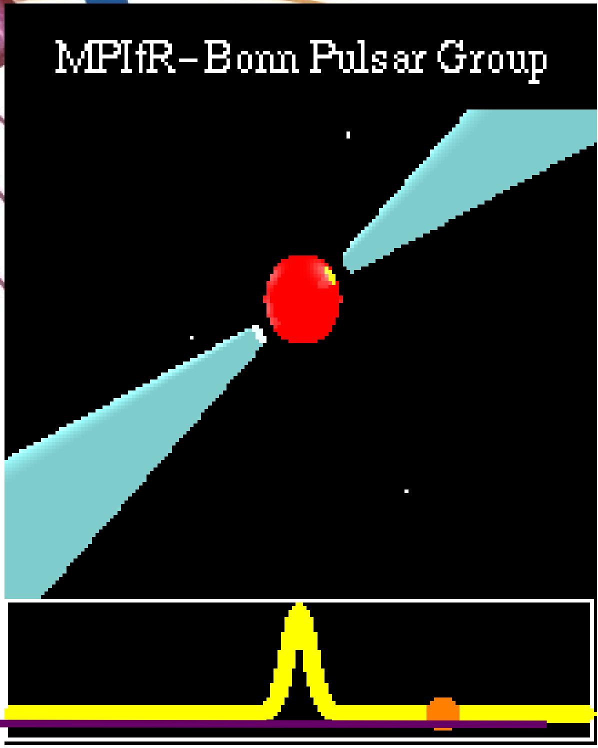
~ 7×10^{-16} J

~ 4000 eV

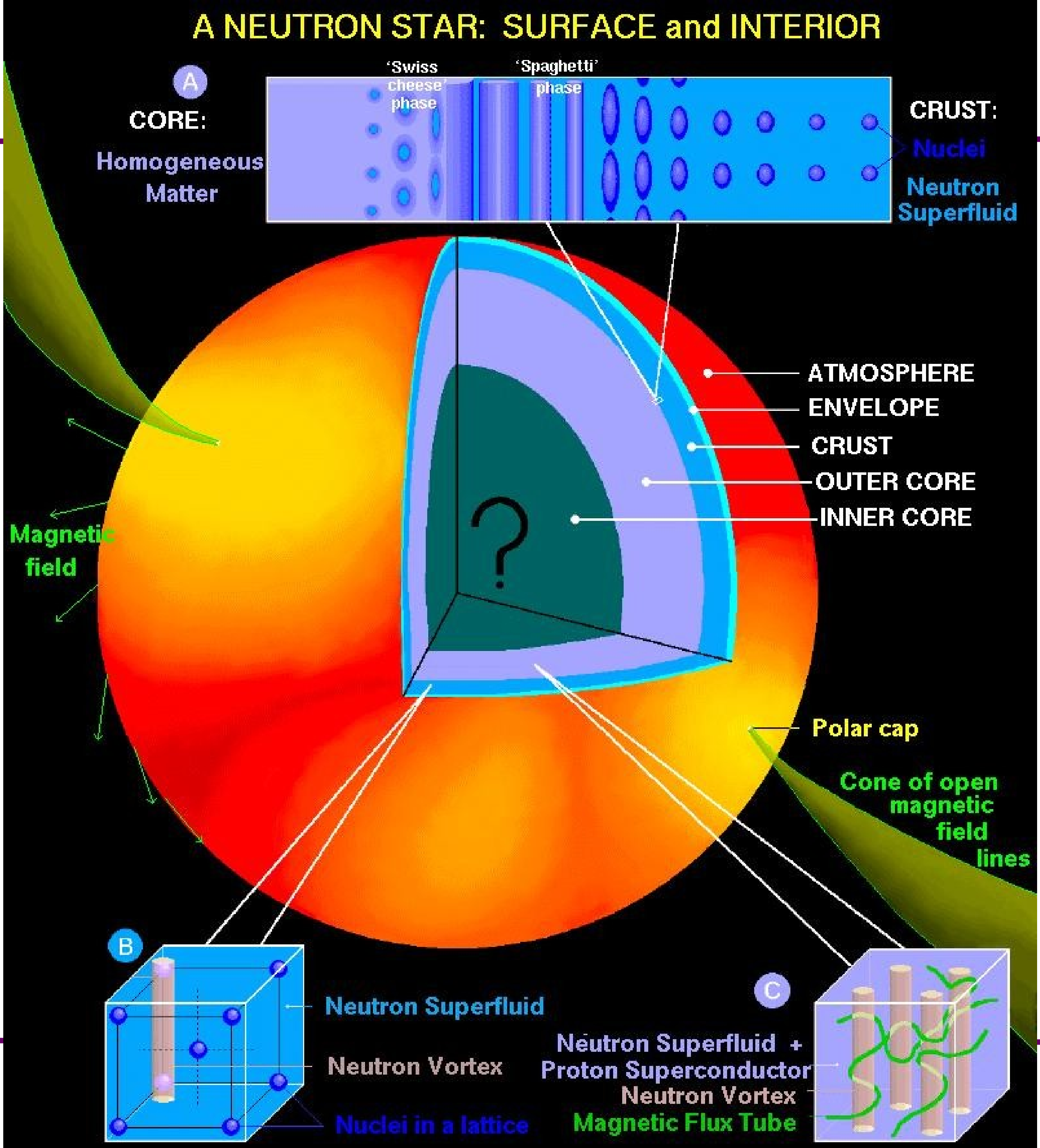
⇒Thermal X-ray radiation







Neutron star interiors



Neutron star interiors

White dwarfs to neutron stars:

ρ_{labo} — 10^4 — 10^7 — $3.2 ? 10^{11}$ — ρ (g/cm³)

No neutronisation

Neutronisation of the nucleii

Neutron drip

$$M(A,Z)+e^- \longrightarrow M(A,Z-1)+n\mathrm{eut}$$

$$p^+ + e^- \longrightarrow n + n_{\mathrm{eut}}$$

- Provokes catastrophic energy loss from system (absorption and emission of neutrinos)

- Collapse of matter in this regime

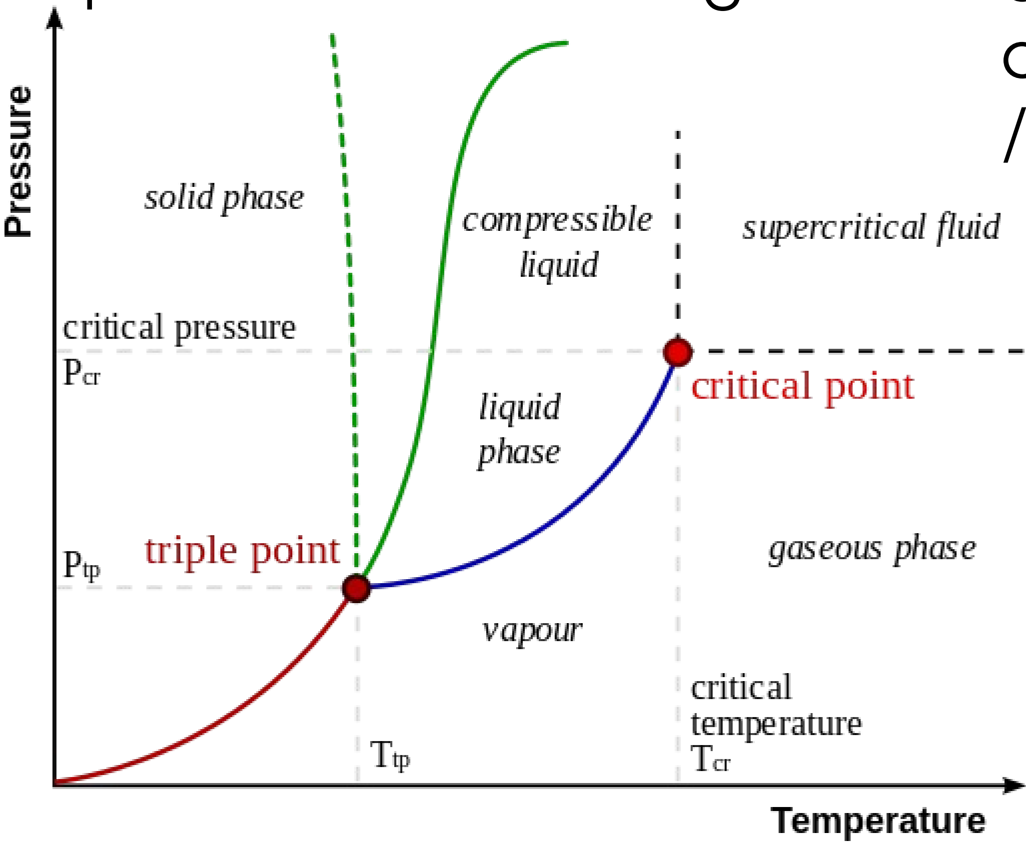
$$M(A,Z) ? \longrightarrow M(A-1,Z)+n$$

- Allows free and stable neutrons

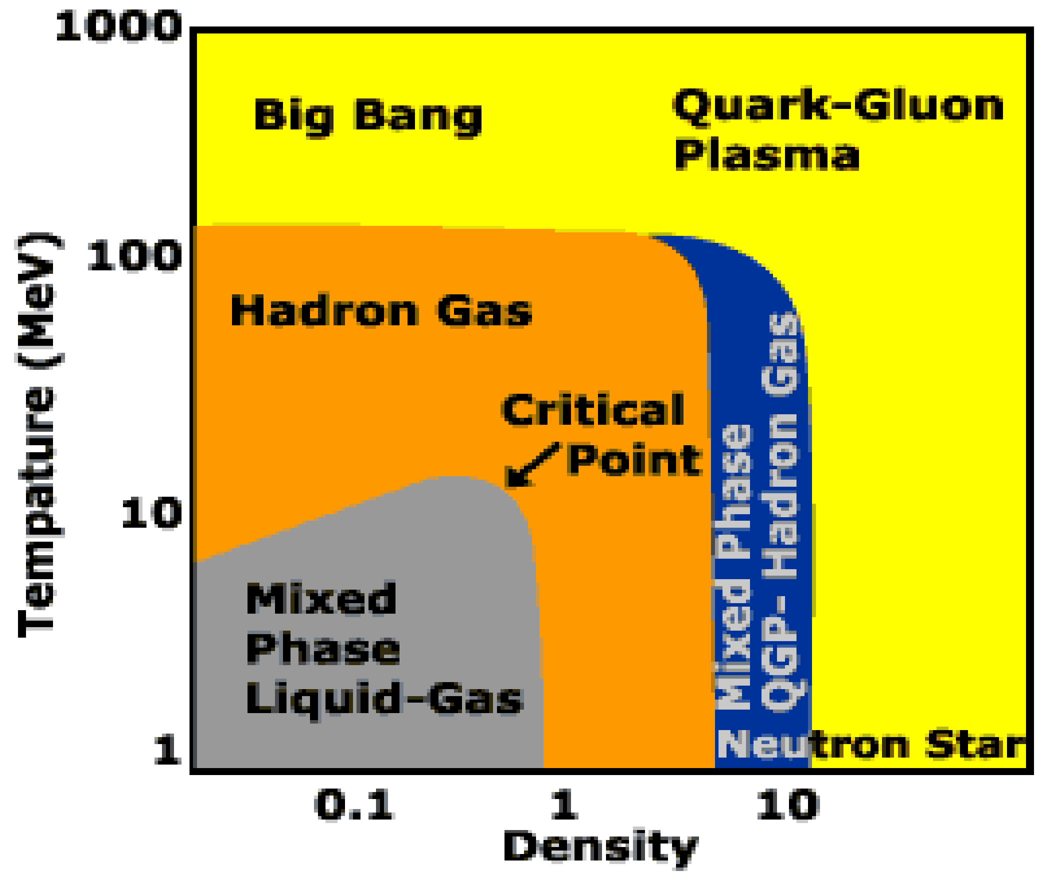
- Stops further collapse

Neutron star interiors : the equation of state

Equation of state of e.g. water



To determine the equation of state of nucleii we need to explore all temperatures and pressures /densities

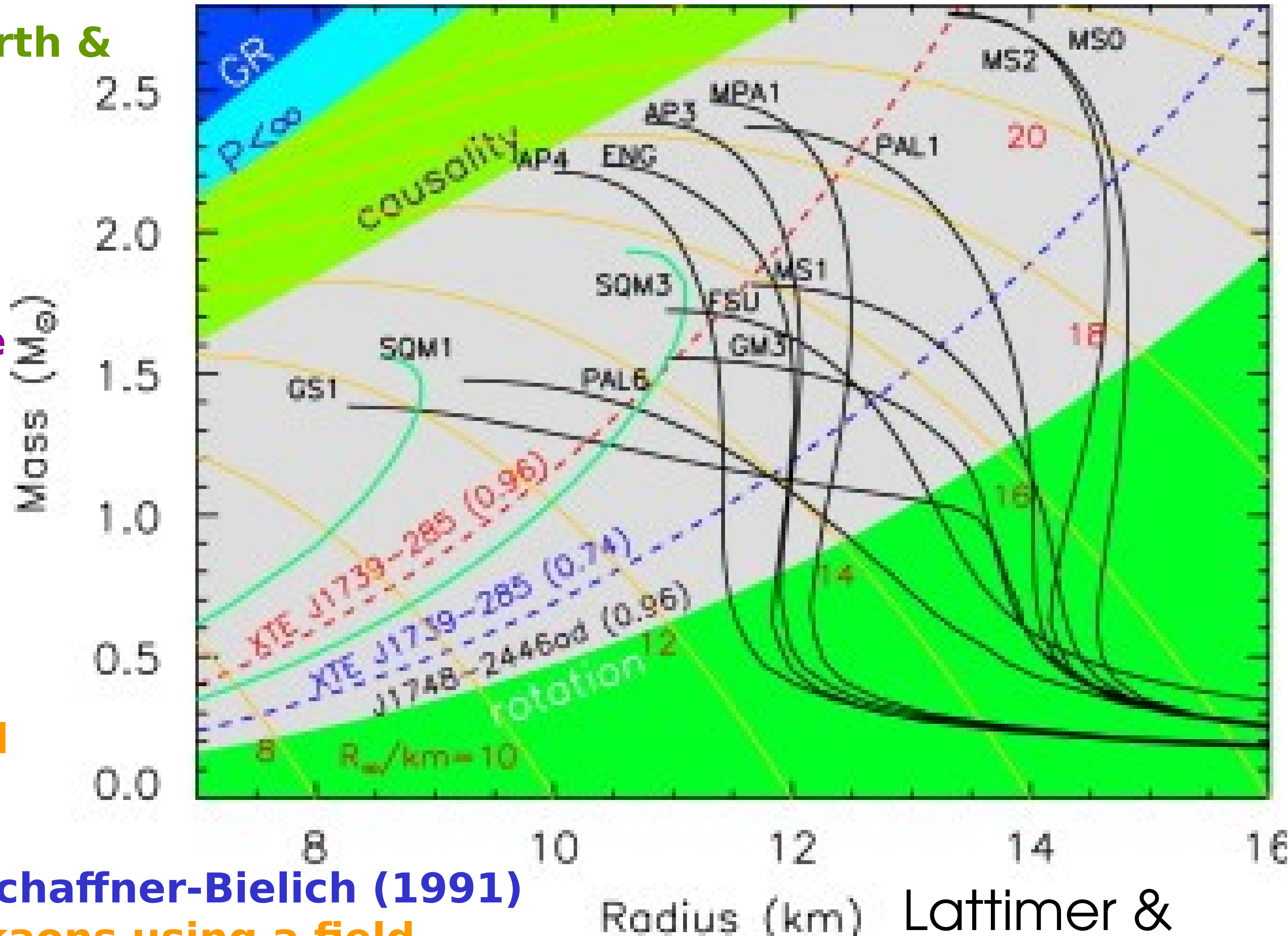


Neutron star interiors : the equation of state

PAL -Prakash, Ainsworth & Lattimer (1988)
Neutrons + protons using a schematic potential

SQM – Prakash, Cooke & Lattimer (1995)
Strange Quark Matter model

GM – Glendenning & Moszkowski (1991)
Neutrons, protons + hyperons using a field theoretical approach



GS – Glendenning & Schaffner-Bielich (1991)
Neutrons, protons + kaons using a field theoretical approach

Lattimer & Prakash (2007)

Neutron star interiors : the equation of state

To determine the neutron star equation of state, we need to know its mass and radius.

Mesuring the radius of a neutron star is comparable to measuring :

A) the height of the Eiffel Tower from London

B) the height of a house in the USA from France

C) the width of a hair on the moon

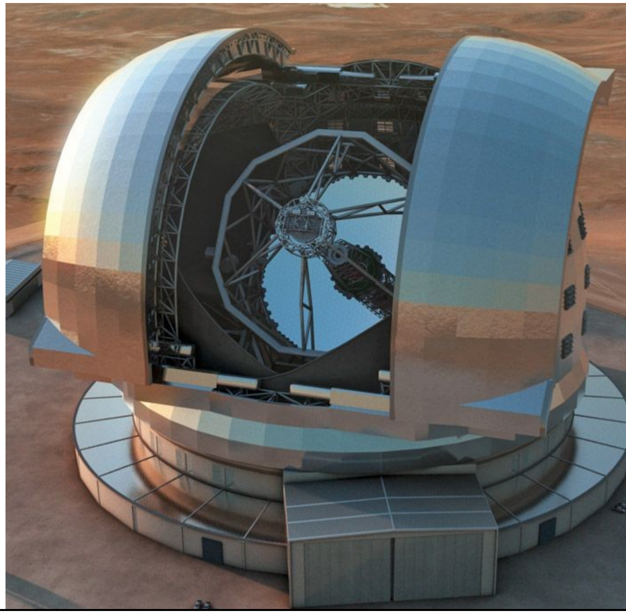
Neutron star interiors : the equation of state

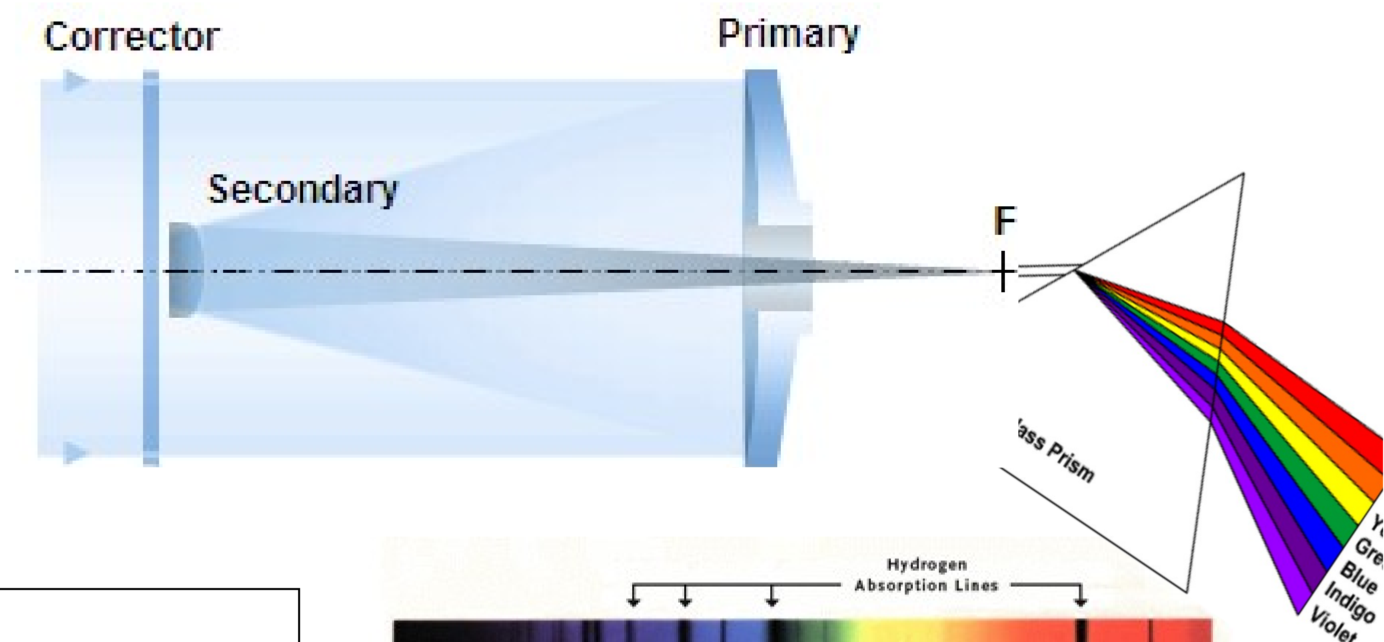
To determine the neutron star equation of state, we need to know its mass and radius.

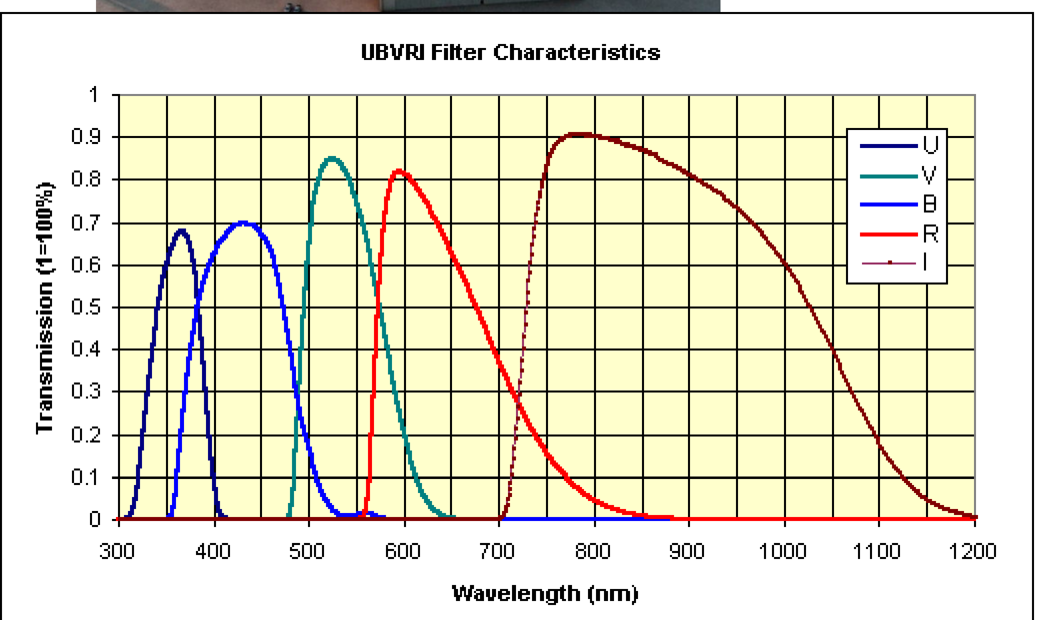
Mesuring the radius of a neutron star is comparable to measuring :

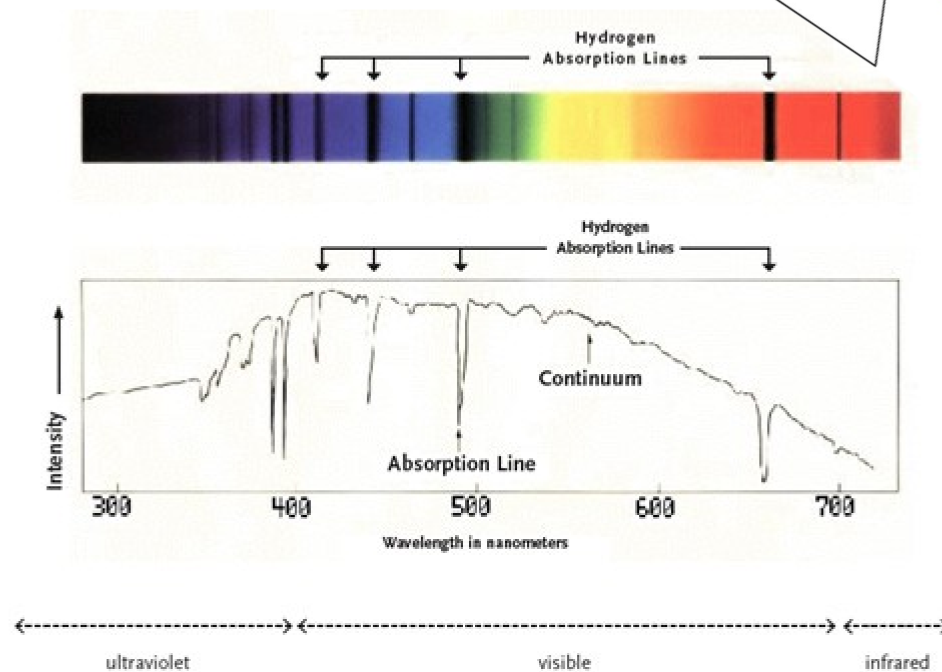
C) the width of a hair on the moon

Neutron star interiors : optical observations



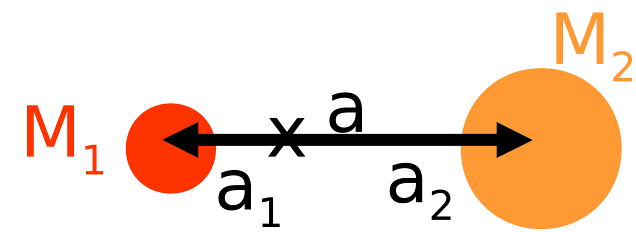






Neutron star interiors : determining the mass

Considering two stars in a binary (1 et 2)
as point like :



The force exerted on each star is equal :

$$|M_1 a_1| = |M_2 a_2| \quad \text{et} \quad \frac{|a_1|}{|a|} = \frac{|M_2|}{|M_1 + M_2|} \qquad (a=a_1+a_2)$$

Taking into account the inclination of the system:

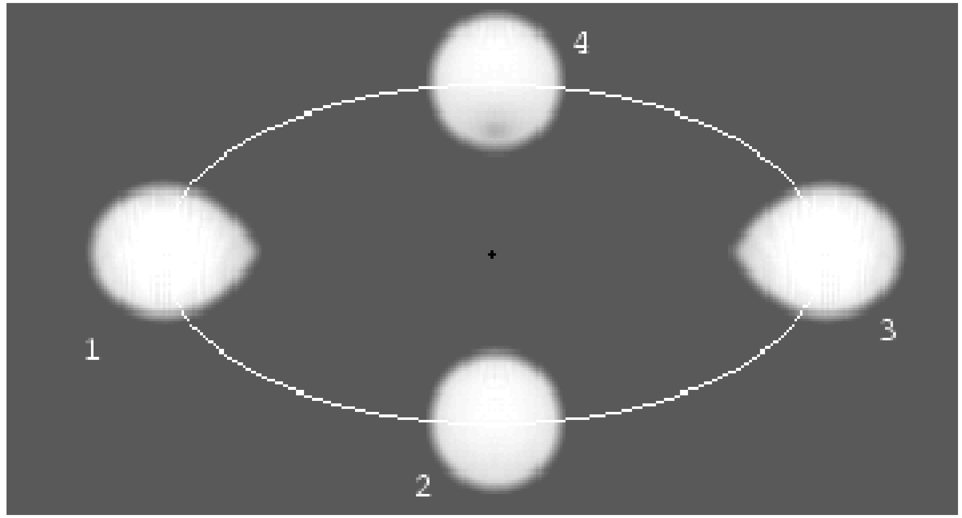
$$(a_1 \sin i)^3 = \frac{M_2^3\, a^3 \sin^3 i}{(M_1 + M_2)^3}$$

And using Kepler's 3rd law :

$$\frac{4p^2 a^3}{P^2} = G(M_1 + M_2) \qquad (a_1 \sin i)^3 = \frac{M_2^3\, G\, P^2 \sin^3 i}{4p\pi^2\,(M_1 + M_2)^2}$$

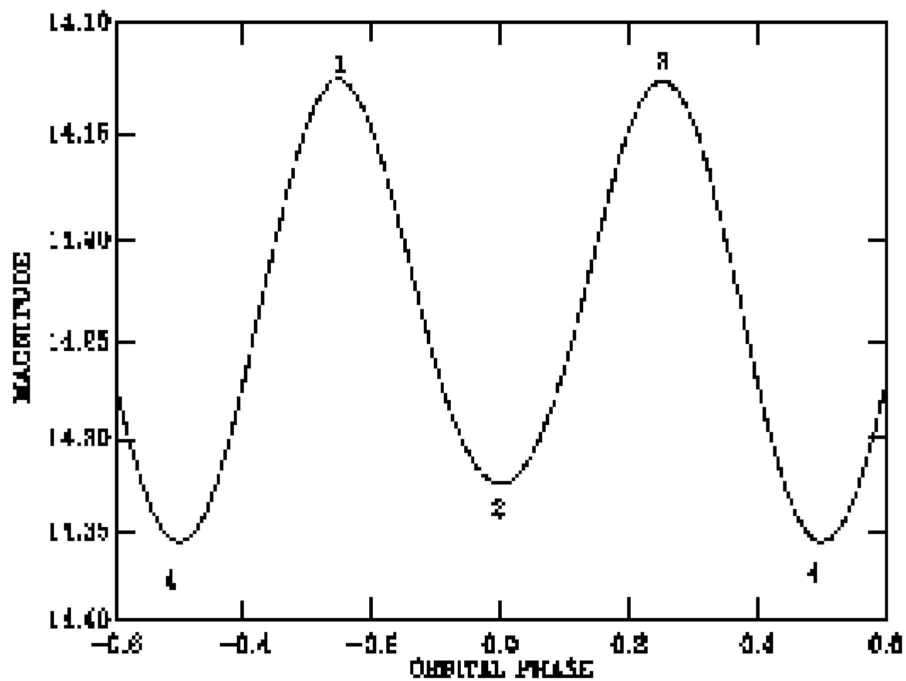
Neutron star interiors

Ellipsoidal modulation

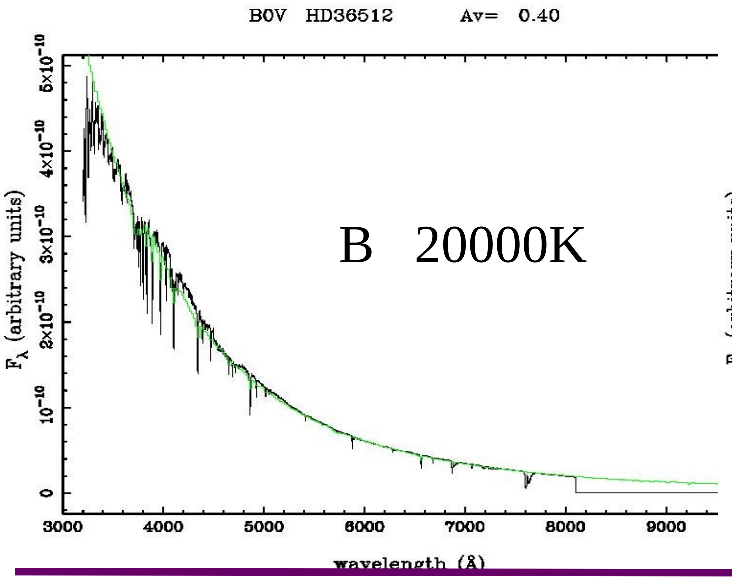


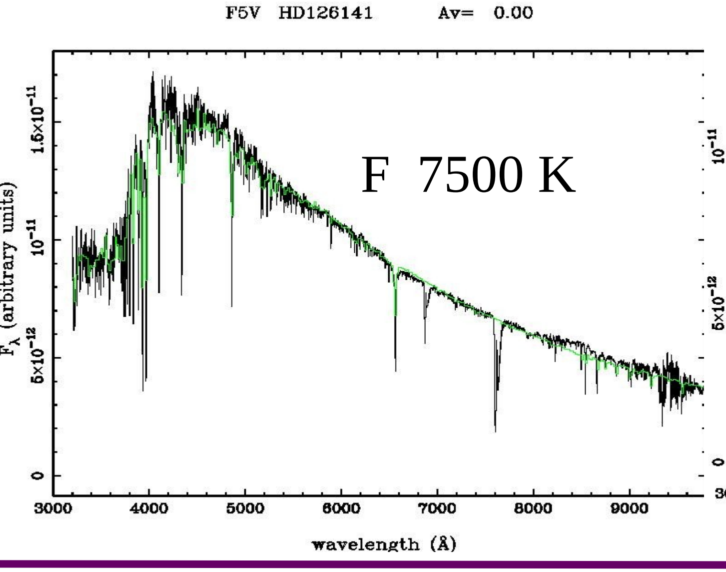
$$F(M) = \frac{M_1^3 \sin^3 i}{(M_1 + M_2)^2}$$

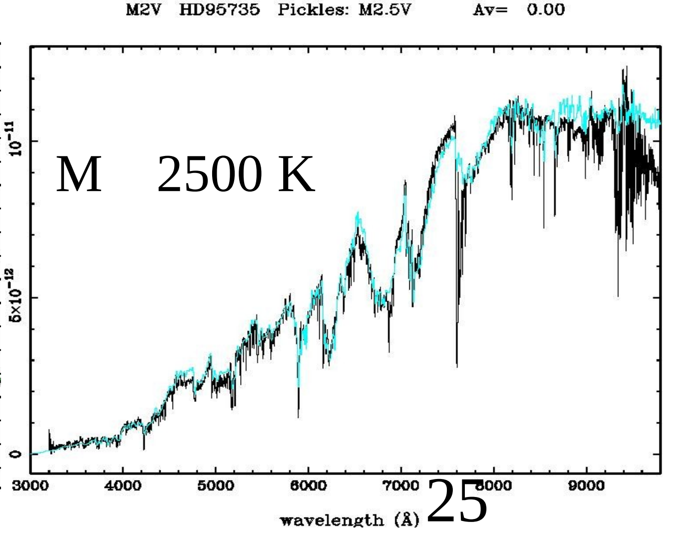
$$= \frac{P K_2^3}{2 \pi G}$$



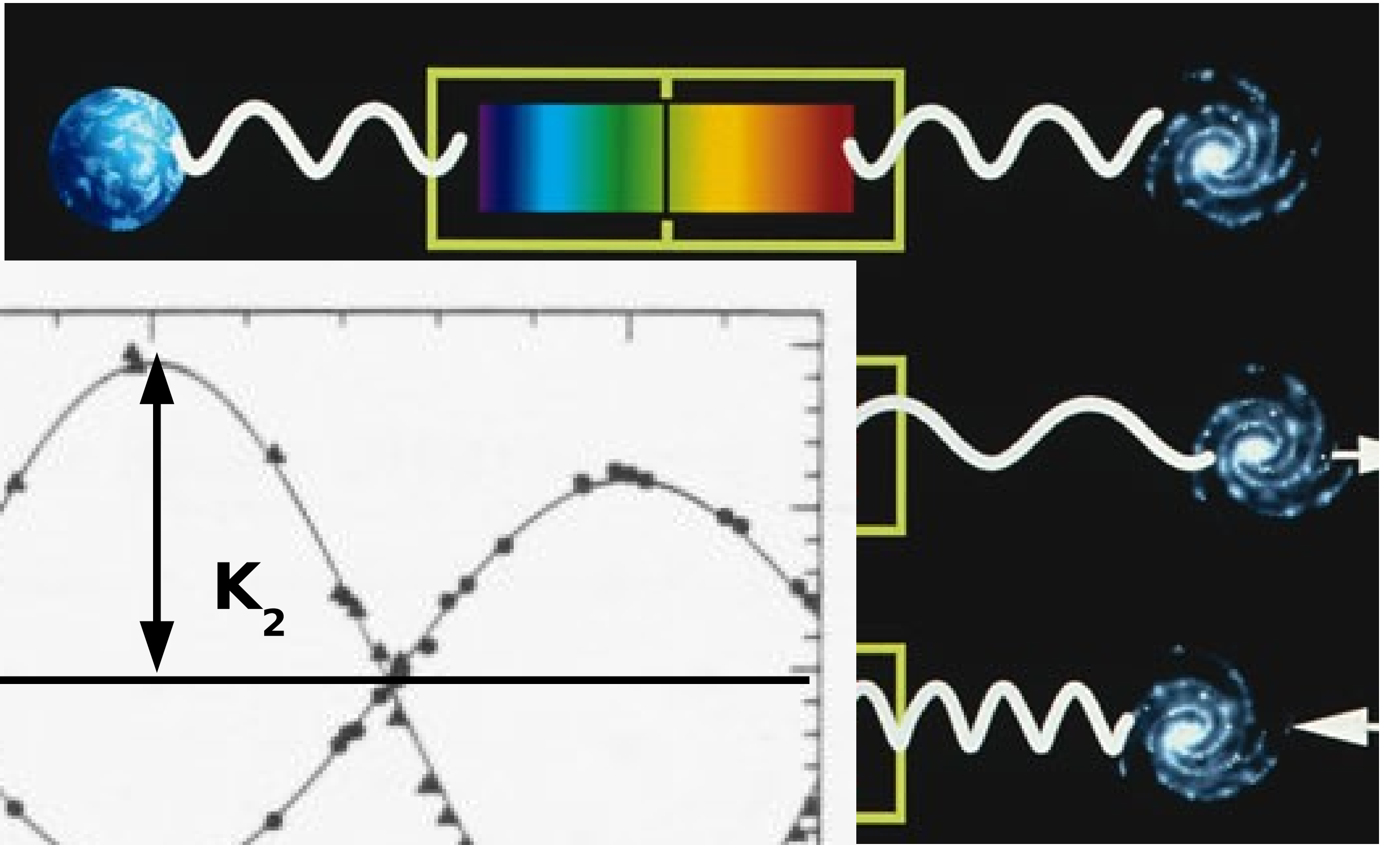

O B A F G K M
'Oh Be A Fine Girl, Kiss Me'
M K G F A B O

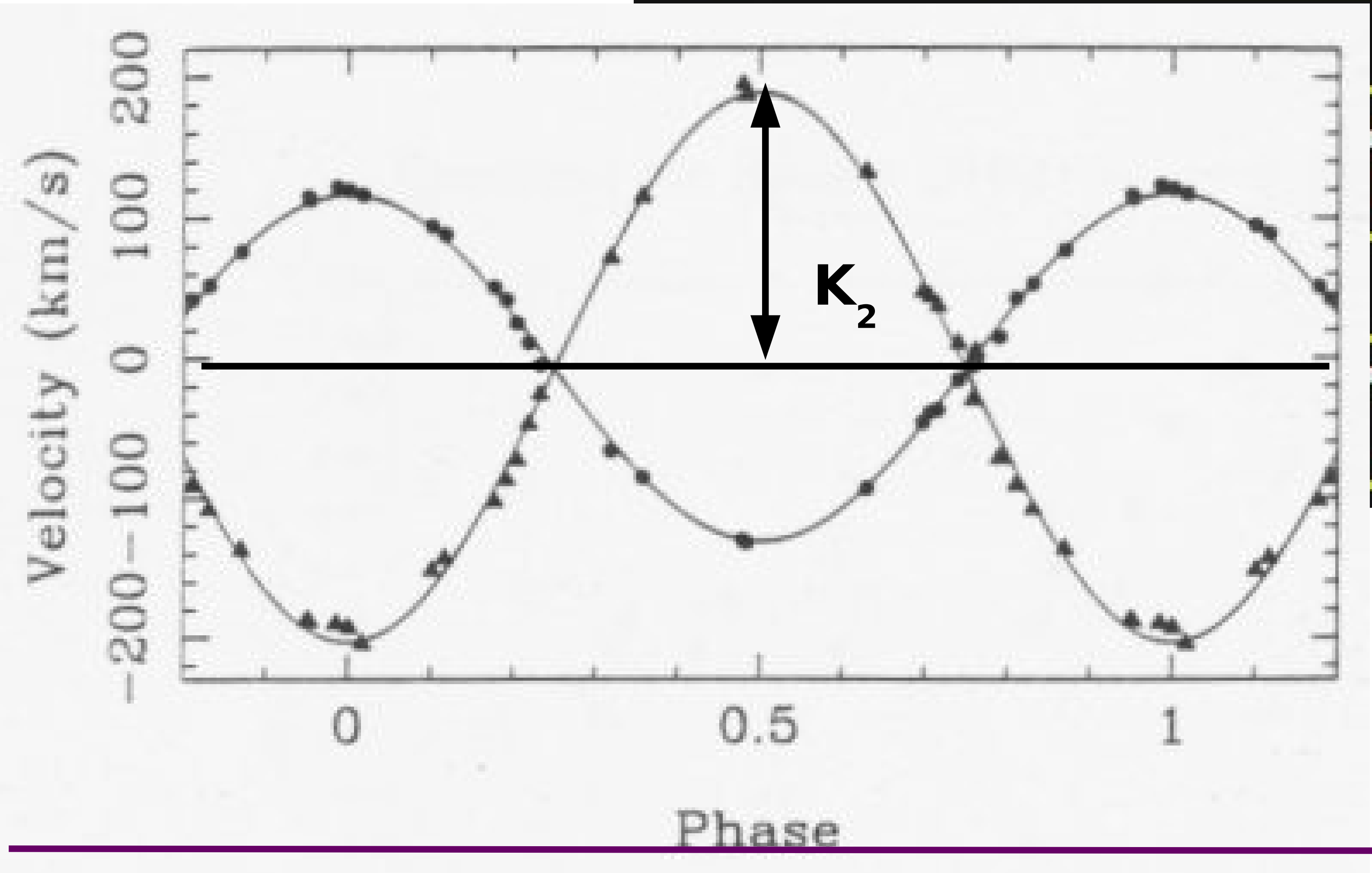

B0V HD36512 Av= 0.40
B 20000K
F_λ (arbitrary units)
wavelength (Å)


F5V HD126141 Av= 0.00
F 7500 K
F_λ (arbitrary units)
wavelength (Å)

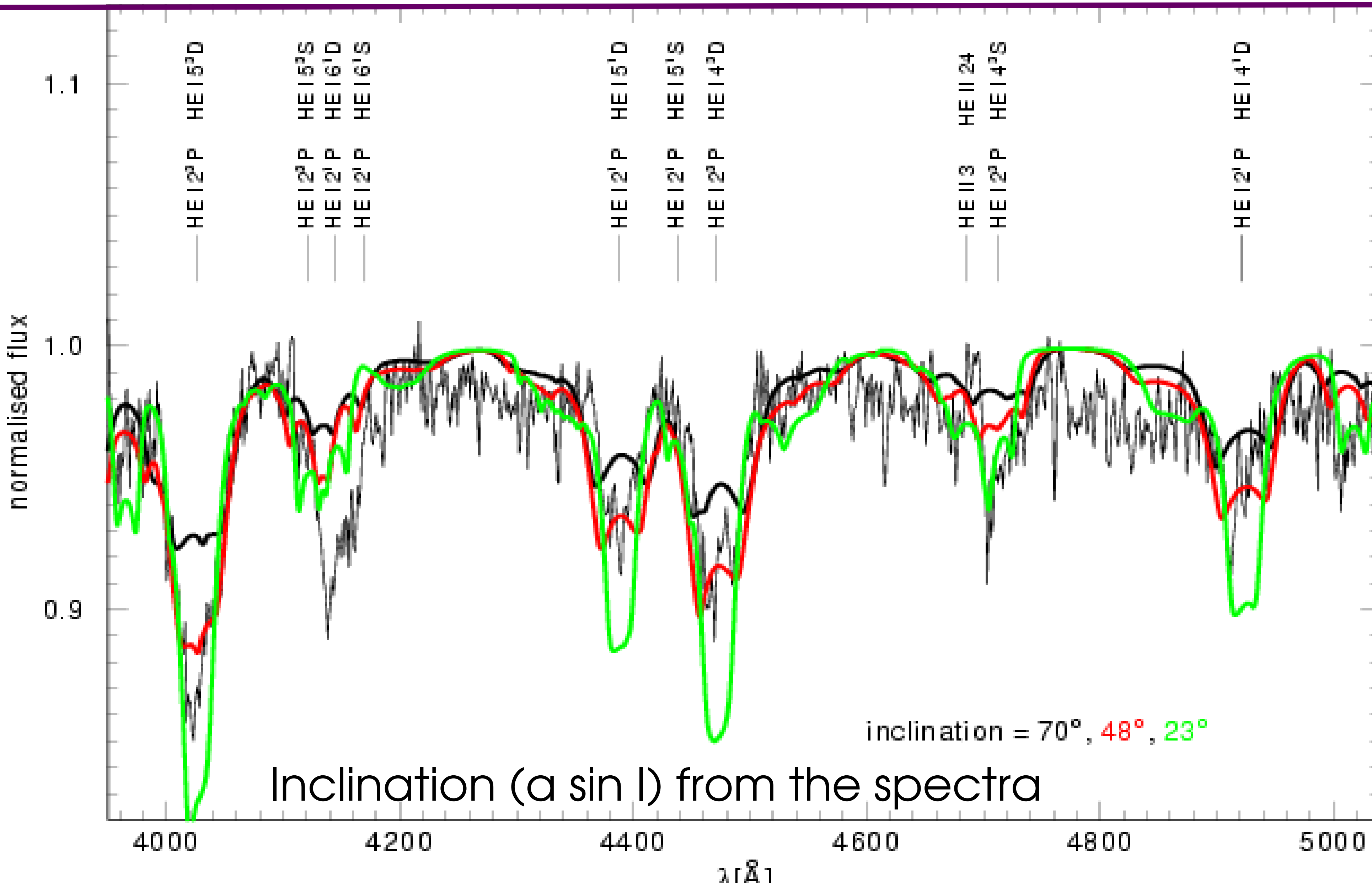

M2V HD95735 Pickles: M2.5V Av= 0.00
M 2500 K
wavelength (Å)

The Doppler effect



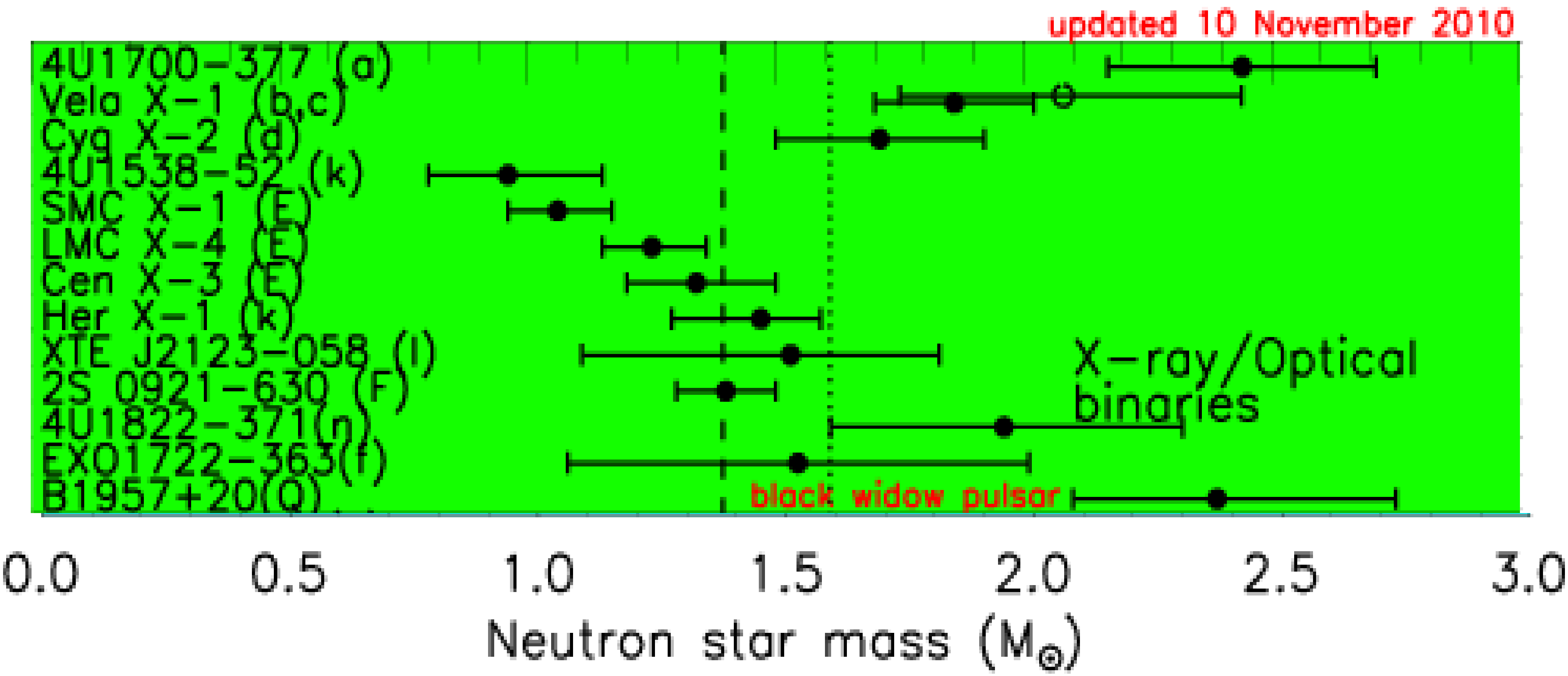


Neutron star interiors



Neutron star interiors

Mass estimates

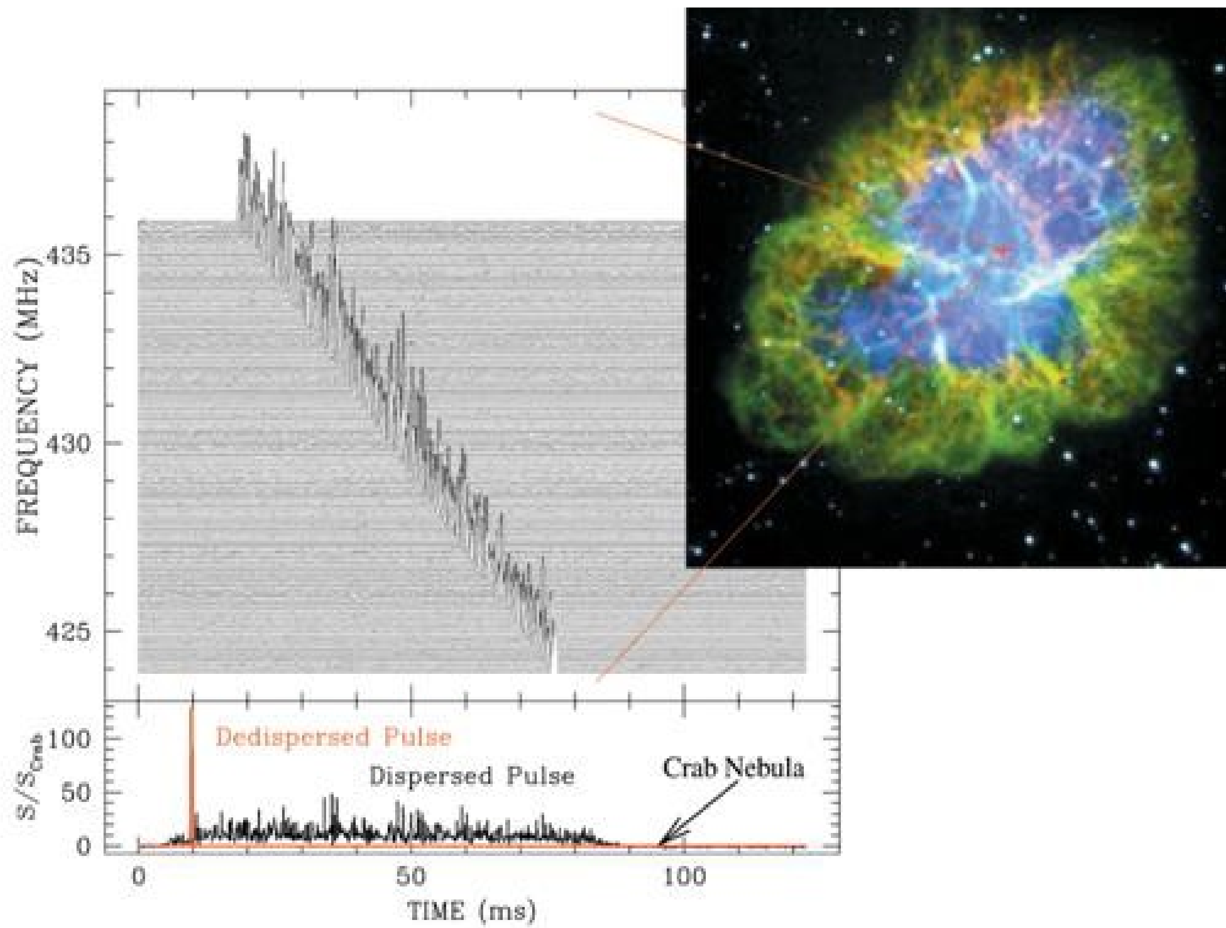


Lattimer & Prakash (2010)

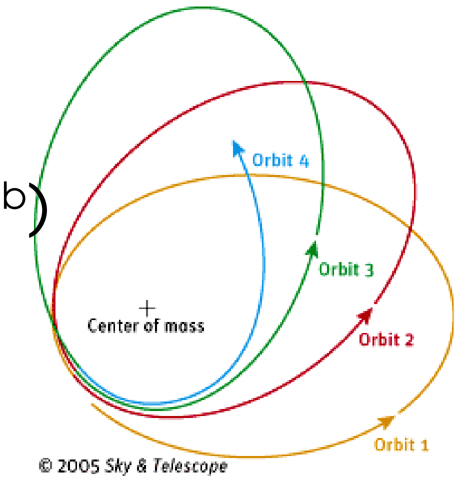
Neutron star interiors : radio observations

$$\theta = 1.220\frac{\lambda}{D}$$

Θ = angular resolution
Λ = wavelength
D = diameter

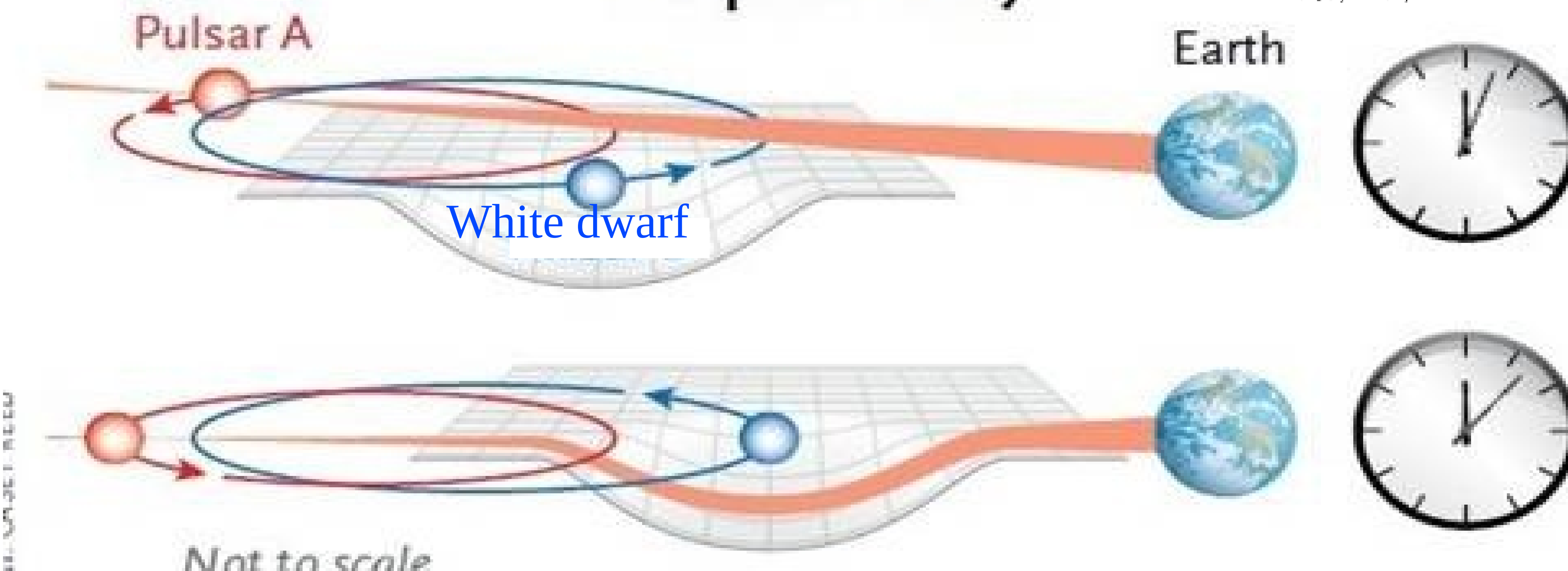


Neutron star interiors



Mean rate of periastron advance ($\dot{\omega}$)
Redshift/time dilation (γ)
Orbital period derivative caused by gravity waves ($\dot{P}_b$)
Range and shape of the Shapiro time delay (r, s)
(due to gravitational time dilation)
R = mass ratio

Shapiro Delay



Neutron star interiors

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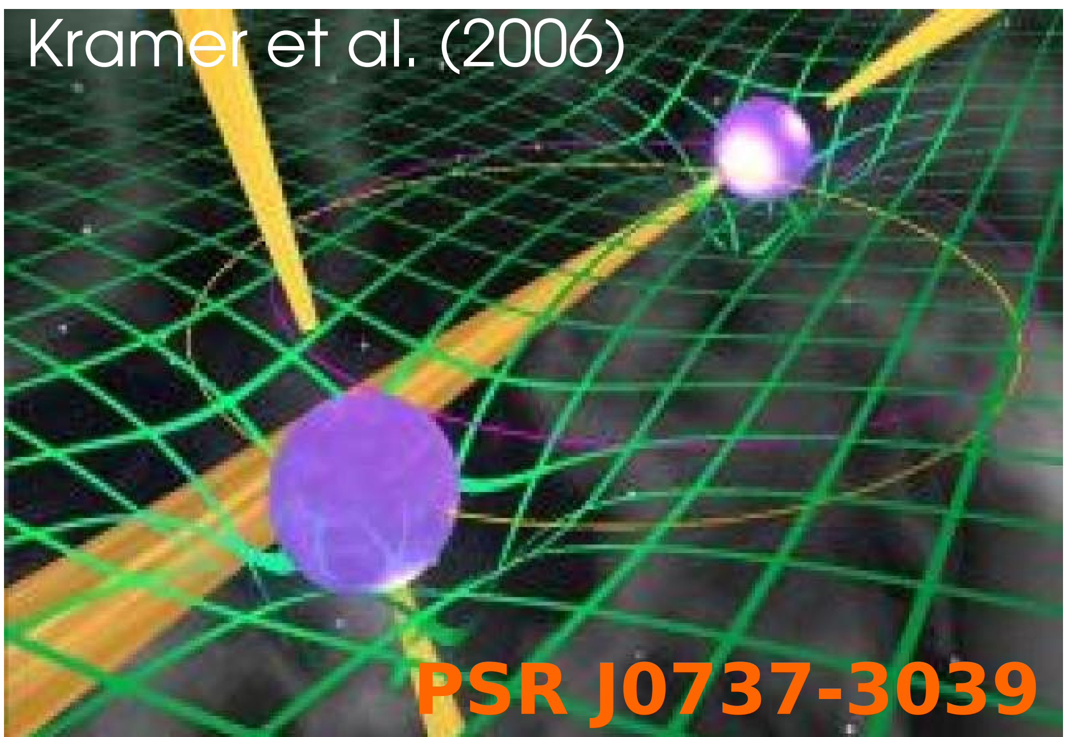
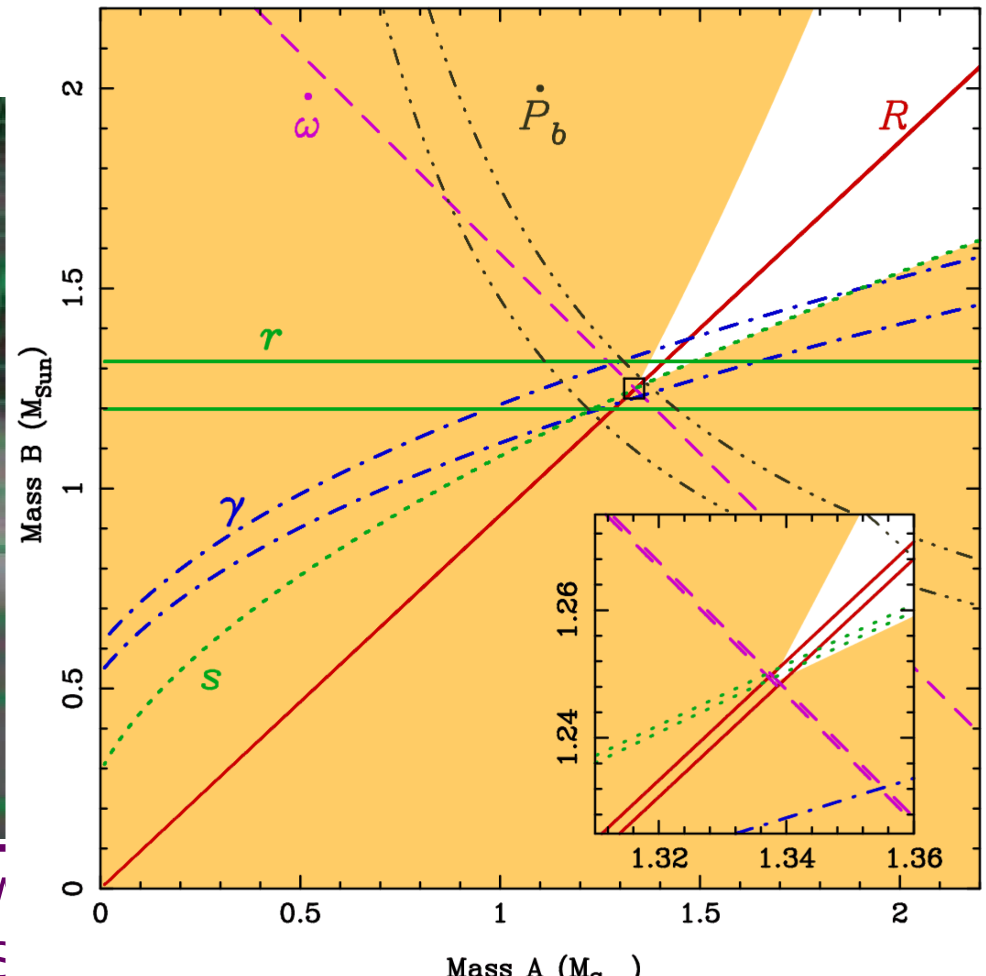
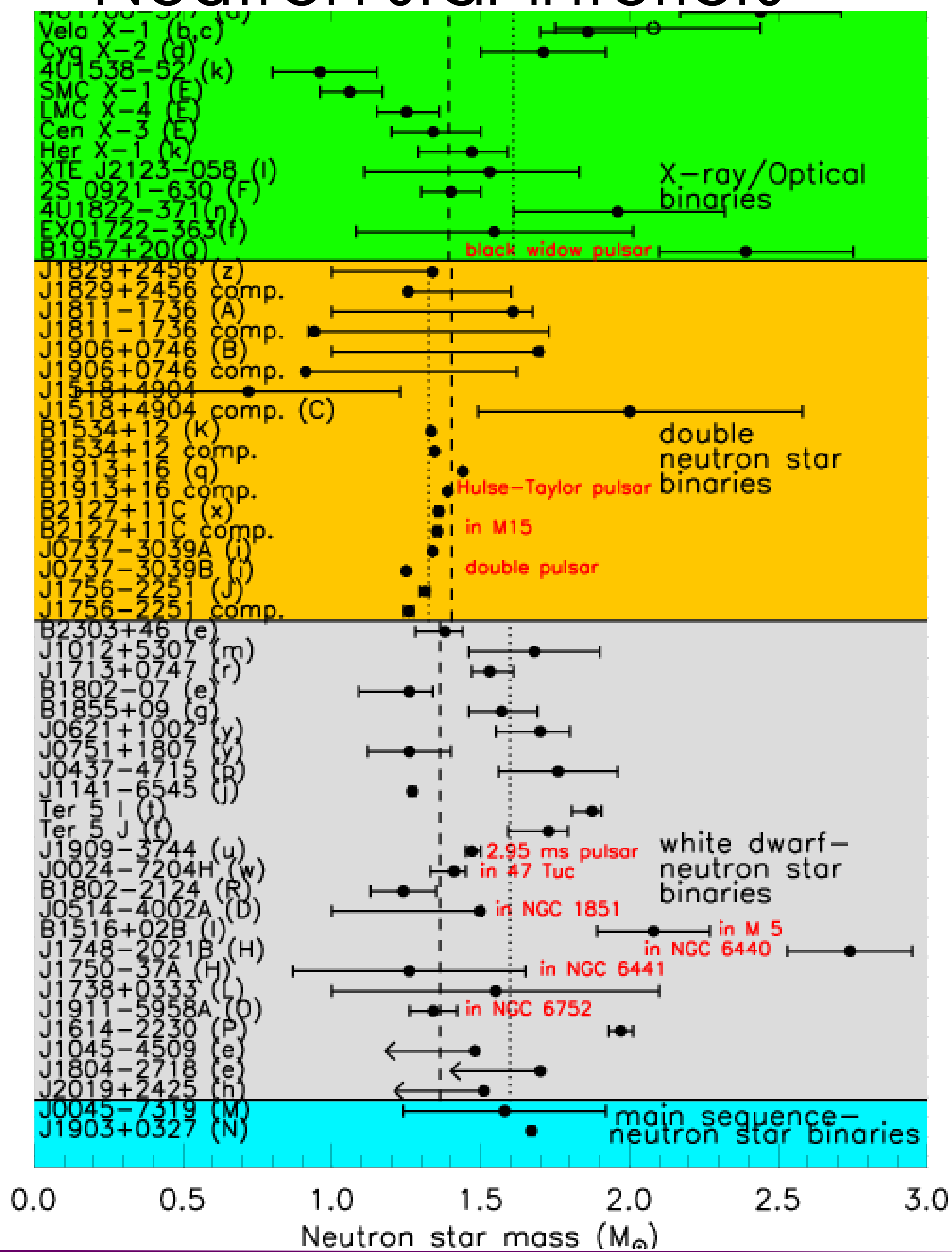


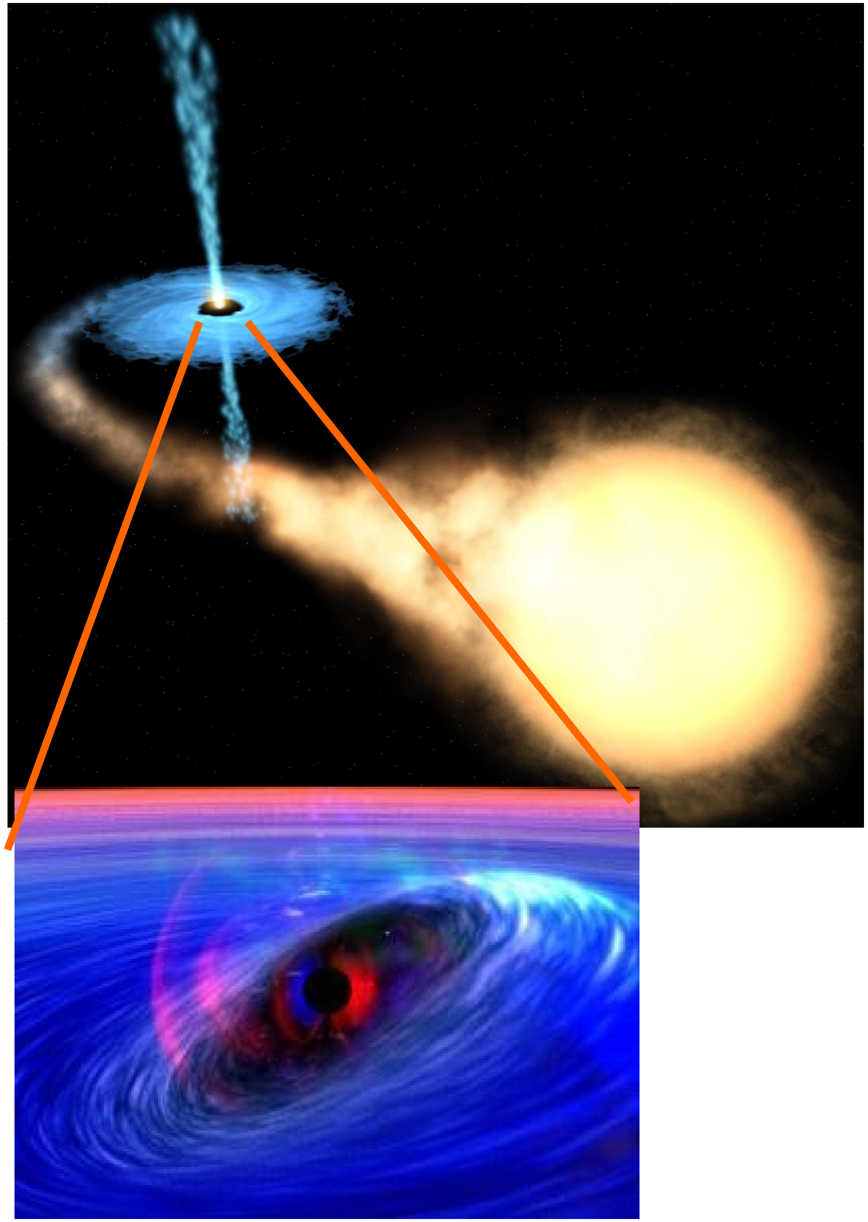


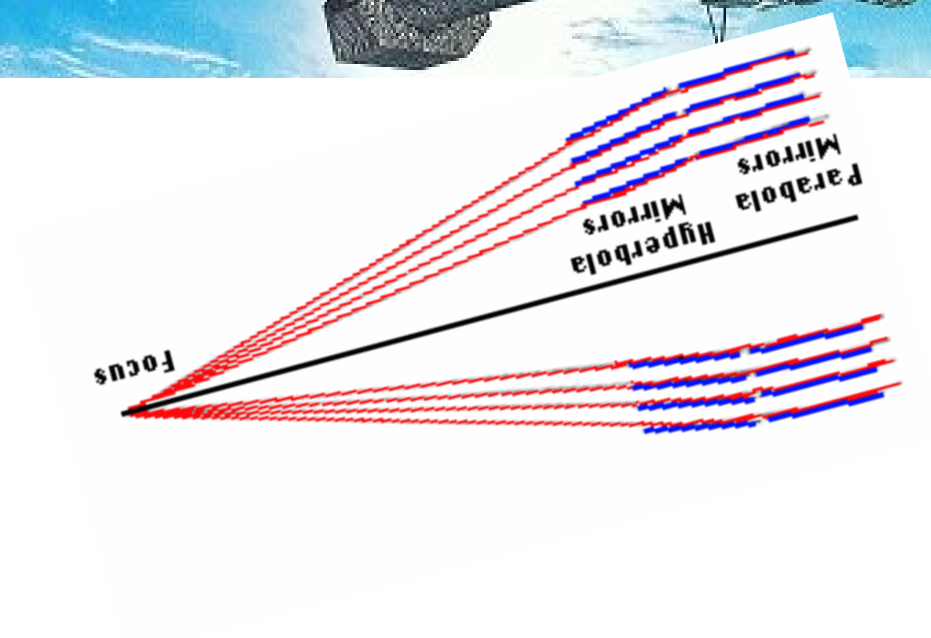
Neutron star interiors



Lattimer & Prakash (2010)

Neutron star interiors : X-ray observations





Neutron star interiors radii from QPOs

Equation of motion for a particle in a Schwarzschild metric:

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{eff}(r) = \frac{\epsilon^2 - c^4}{2c^2}$$

$$V_{eff}(r) = -\frac{GM}{r} + \frac{l^2}{2r^2} - \frac{GMl^2}{c^2 r^3}$$

l : kinetic energy per unit mass

r : Orbital radius

ϵ : Measured energy per unit mass of the particle

$= c^2(1-R_s/c^2)dt/d\tau$

$$\tau_{dyn} = \left(\frac{r^3}{GM}\right)^{1/2} \simeq 0.5 - 10\mathrm{msec}$$

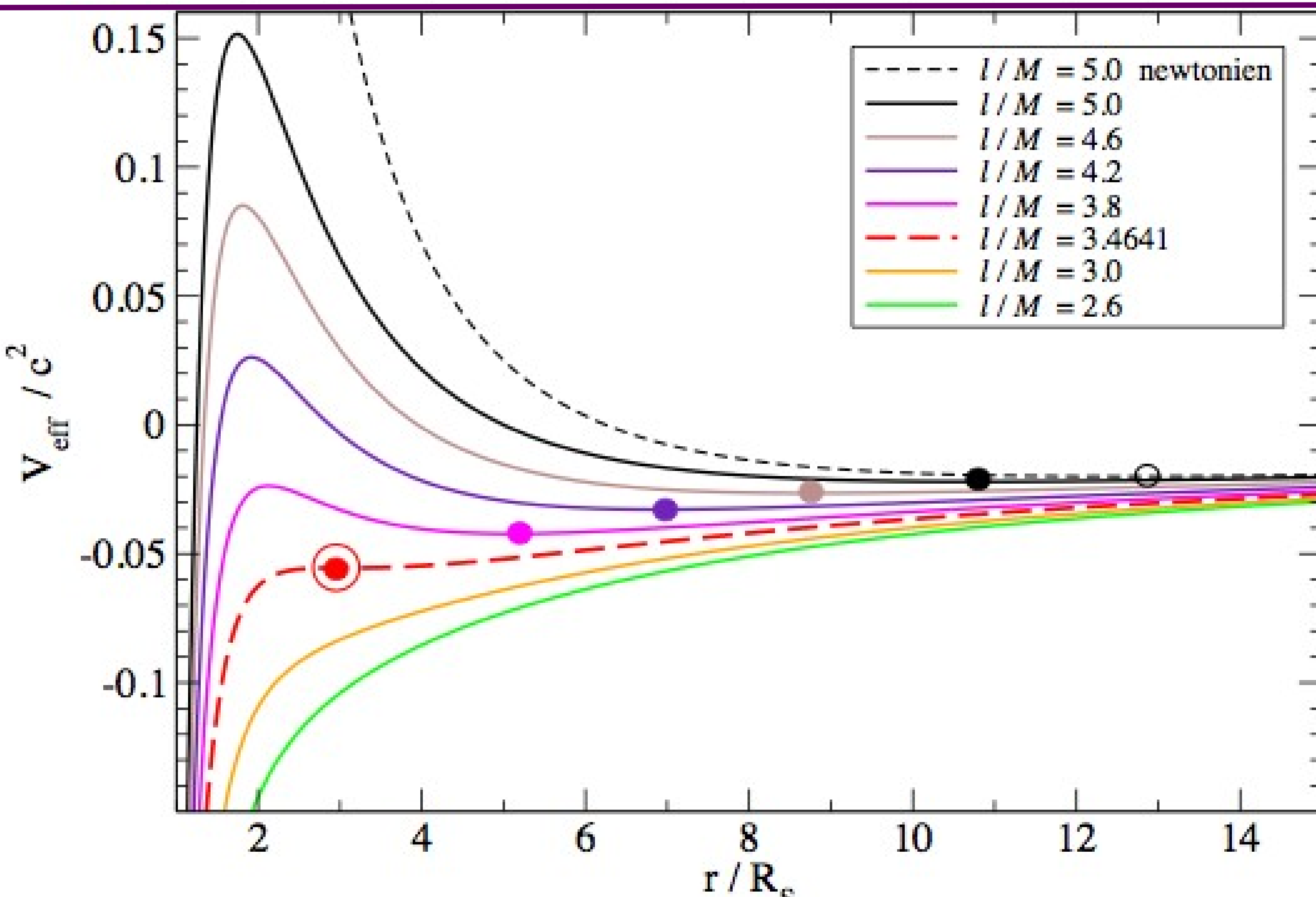
$$l \geq l_{crit} = 2\sqrt{3}\frac{GM}{c}$$

$$l = l_{crit}$$

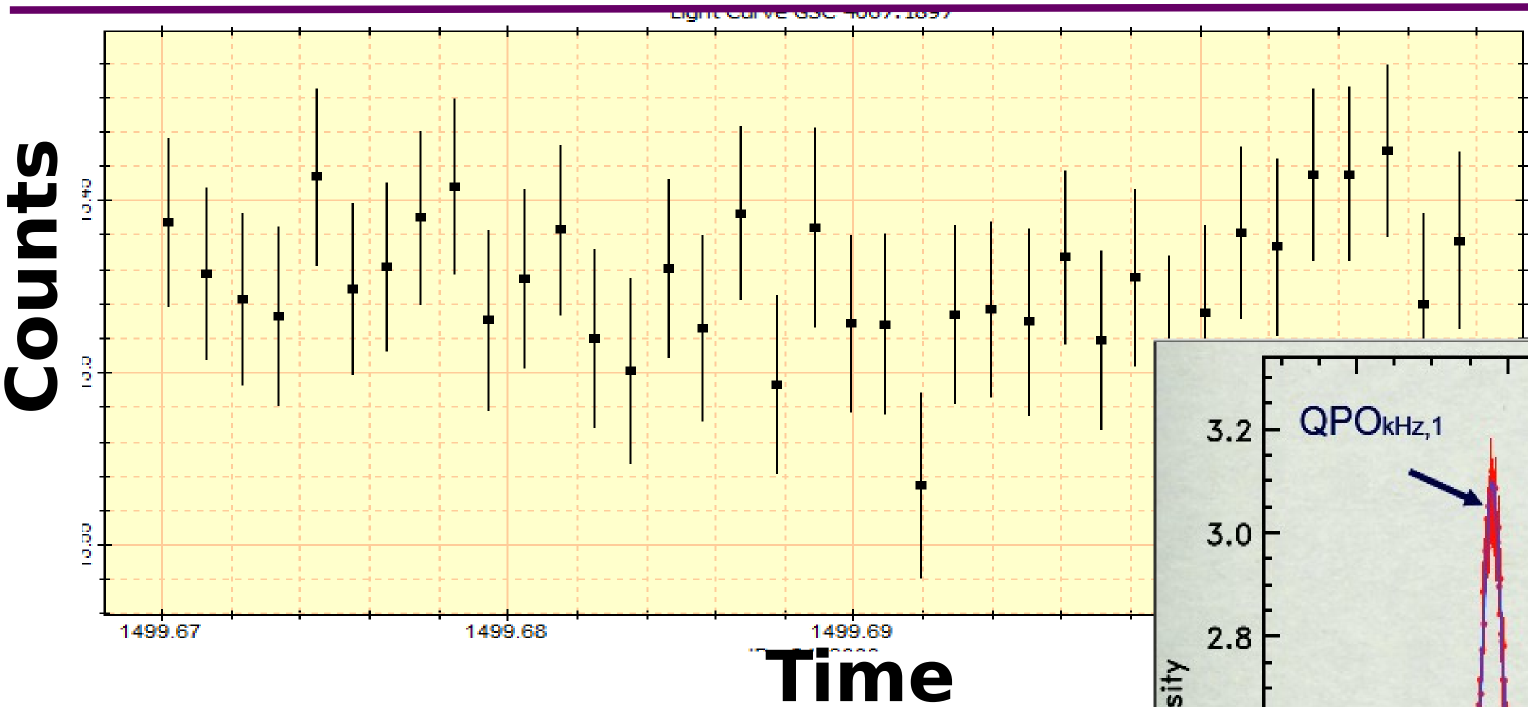
For the innermost stable circular orbit:

$$r_{\mathrm{ISCO}} = 6\frac{\mathrm{GM}}{c^2} = 12.5\mathrm{m}_{1.4}\,\mathrm{km}$$

Neutron star interiors : radii from QPOs

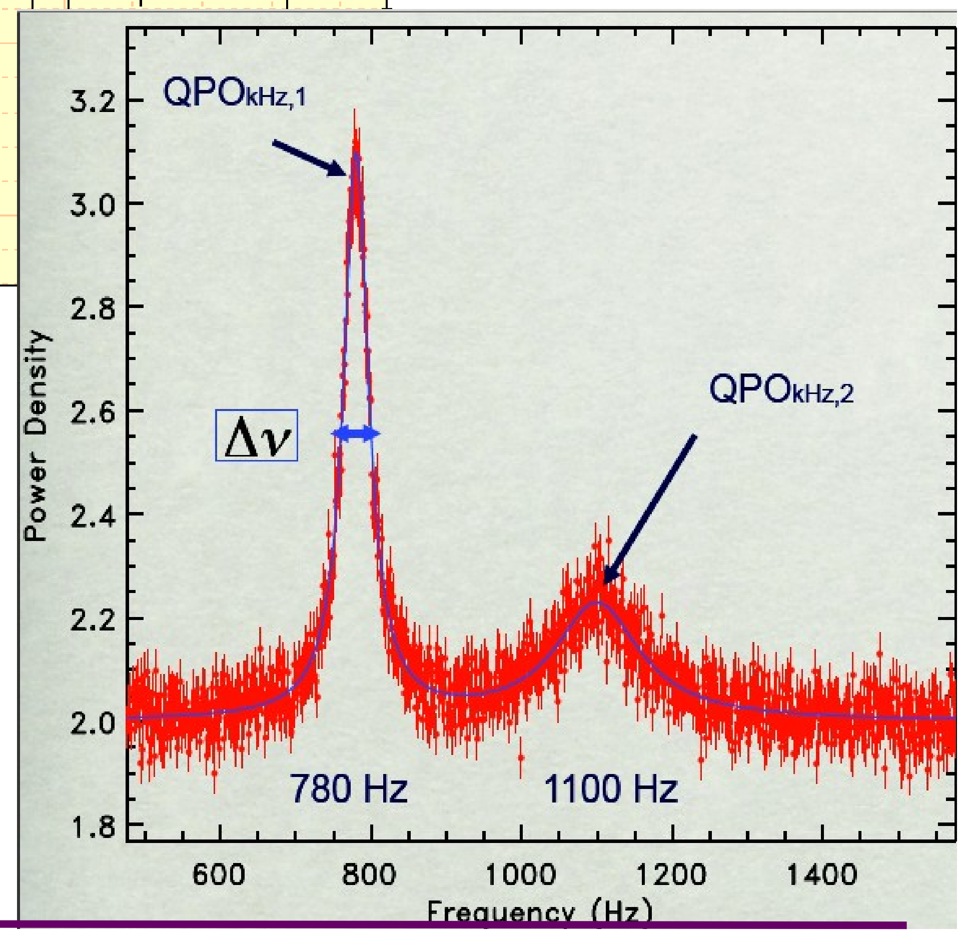


Neutron star interiors : radii from QPOs



X-ray lightcurve

FFT – Fast Fourier Transform



ν – central frequency
Q – quality factor = $\nu/\Delta\nu$

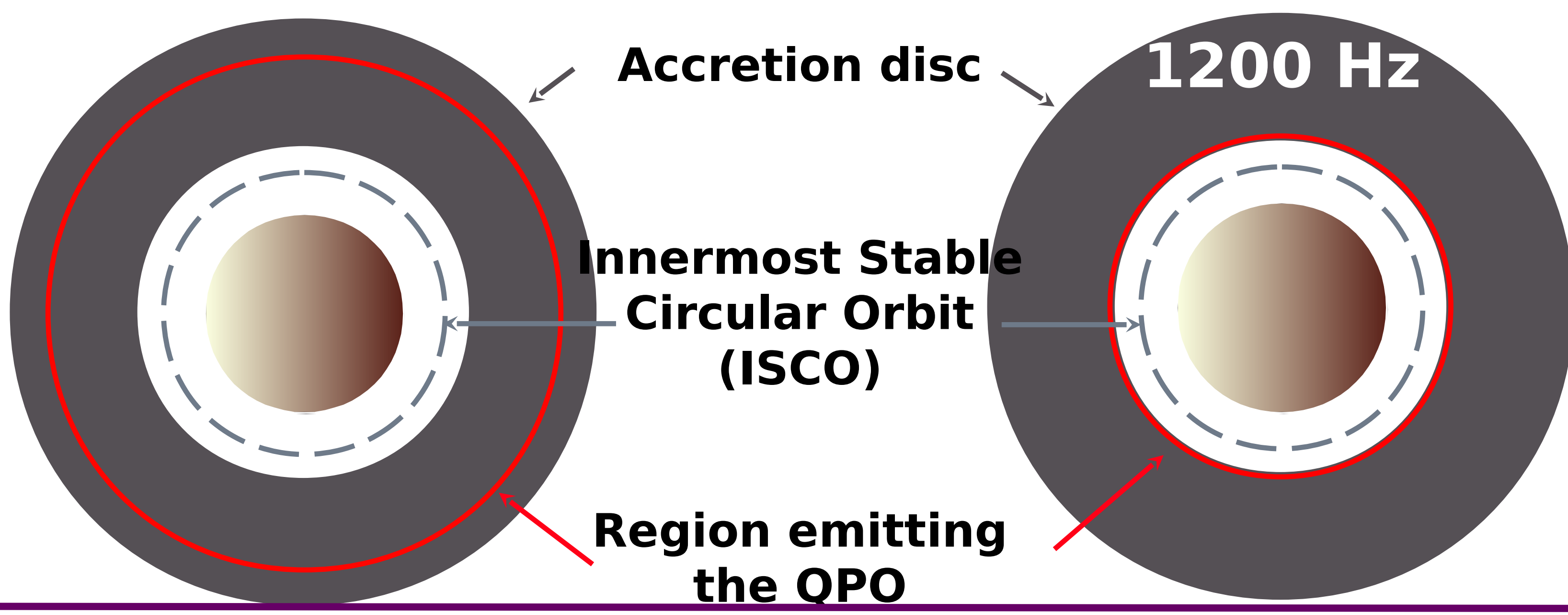
Neutron star interiors : radii from QPOs

Keplerian orbital frequency

$$\nu_{orb} = \left(\frac{GM}{4\pi^2 r_{orb}^3}\right)^{1/2} = 1200\,\mathrm{Hz}\left(\frac{r_{orb}}{15\,\mathrm{km}}\right)^{-3/2} m_{1.4}^{1/2} \Rightarrow \mathbf{QPO_{kHz,2}}$$

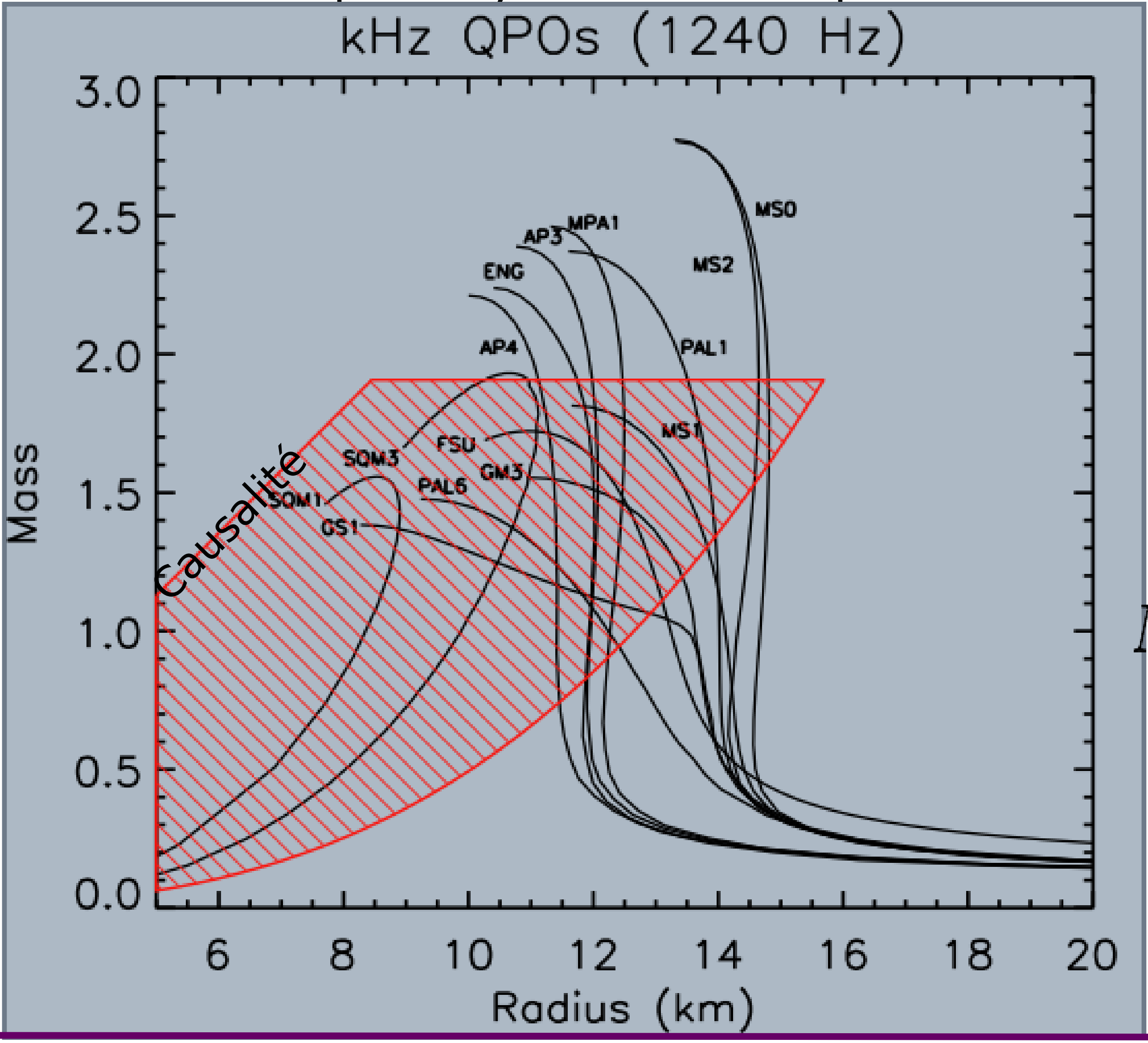
$$\nu_{beat} = \nu_{orb} - \nu_{spin} \Rightarrow \mathbf{QPO_{kHz,1}}$$

500 Hz



Neutron star interiors : radii from QPOs

Maximal frequency and the equation of state



Causality limit

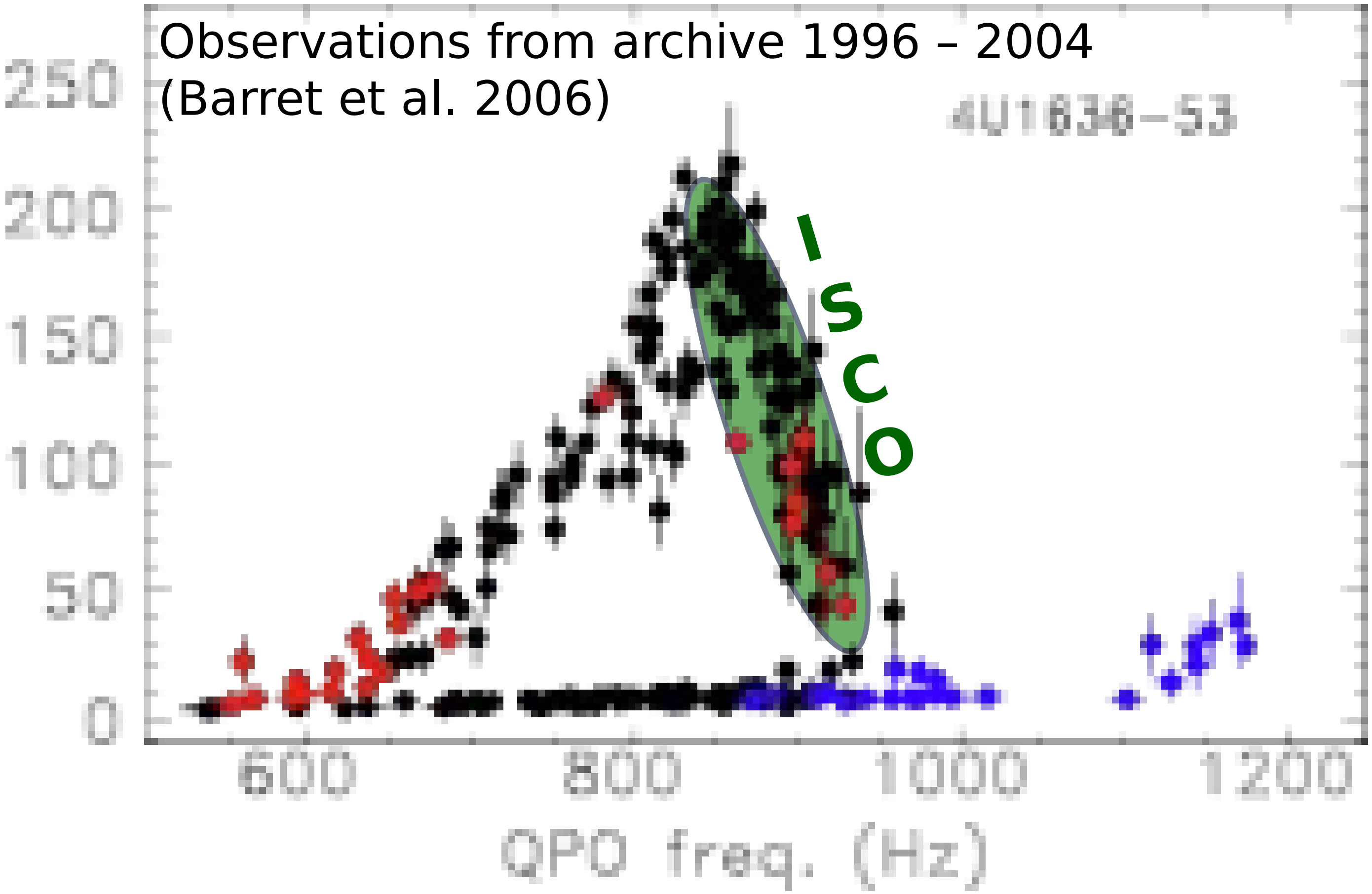
$$M < \frac{c^2 R}{2.9G}$$

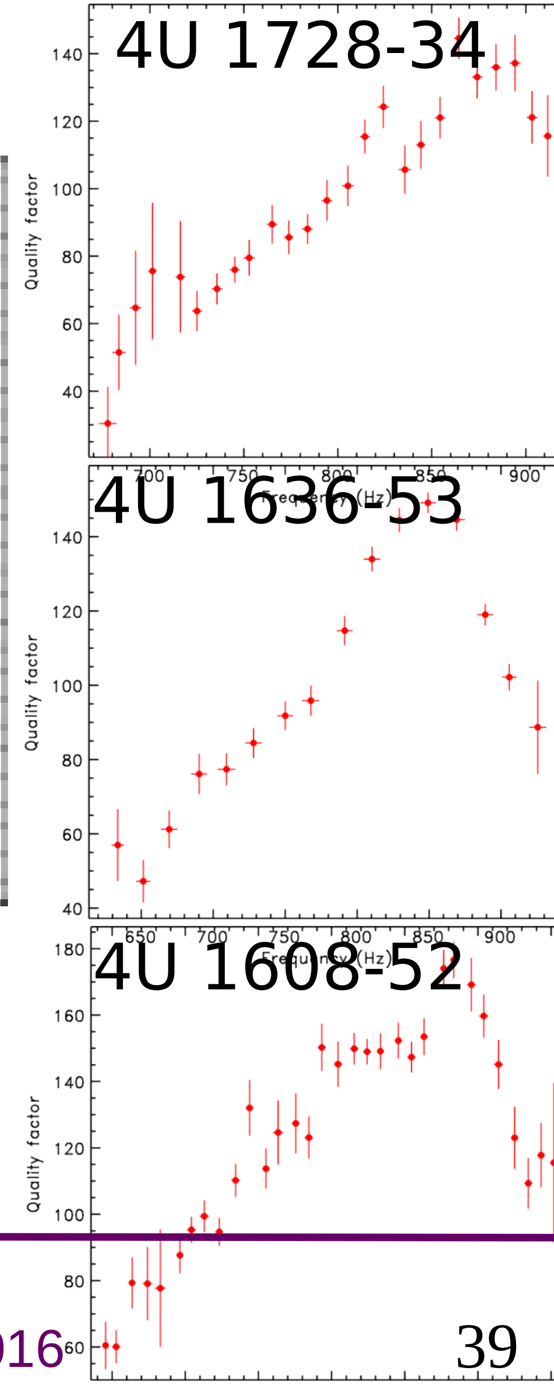
Limits imposed by the QPO :

$$M > \left(\frac{R}{12.9\,\mathrm{km}}\right)^3 \left(\frac{\nu_{QPO}}{1240\,\mathrm{Hz}}\right)^2$$

$$M < 1.95 \frac{1240\,\mathrm{Hz}}{\nu_{QPO}}$$

Neutron star interiors : radii from QPOs





Neutron star interiors : radii from QPOs

Mass of compact object :

Reduction in quality factor : $\nu_{QPOkHz,1max} \approx 920$ Hz

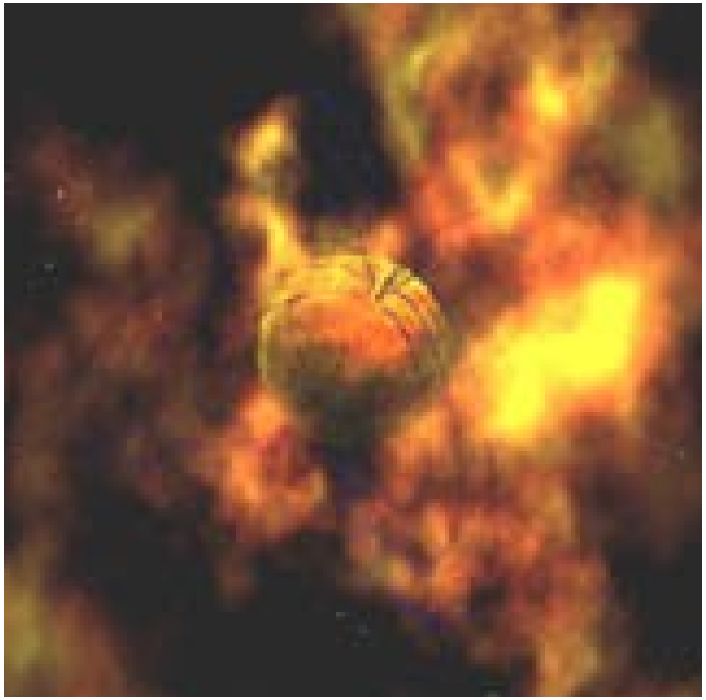
ISCO frequency: $\nu_{QPOkHz,1max} + \Delta\nu \approx 1250$ Hz

$$M \approx 2.2 M_{\odot} \frac{1000\mathrm{Hz}}{\nu_{ISCO}} \times (1 + 0.75 j) \qquad j = \frac{cJ}{GM^2} \sim 0.1 - 0.2$$

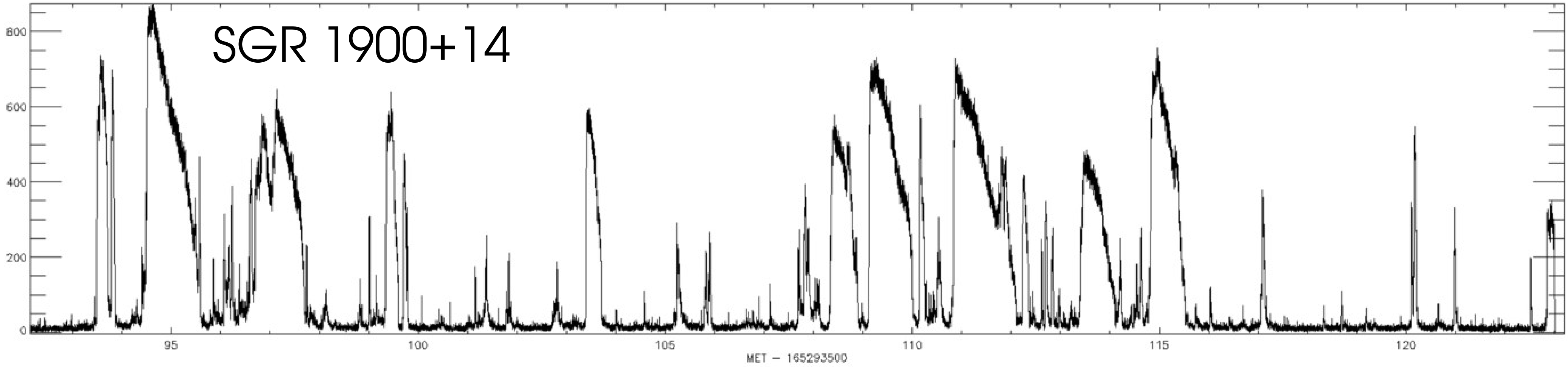
Mass of 4U 0614+09: 1.9-2.1 solar masses

Neutron star interiors: mass & radius from bursts

The mass and radius from gamma-ray repeaters



- Magnetic field (B) ~$10^{14\text{-}15}$G
- B causes huge forces on the solid crust
- -> the crust can rupture (starquake)
- Brief but intense hard X-ray emission



Neutron star interiors: mass & radius from bursts

The Eddington limit

$$L_E = \frac{4\pi\ G\ M\ m_p\ c}{\sigma_0}$$

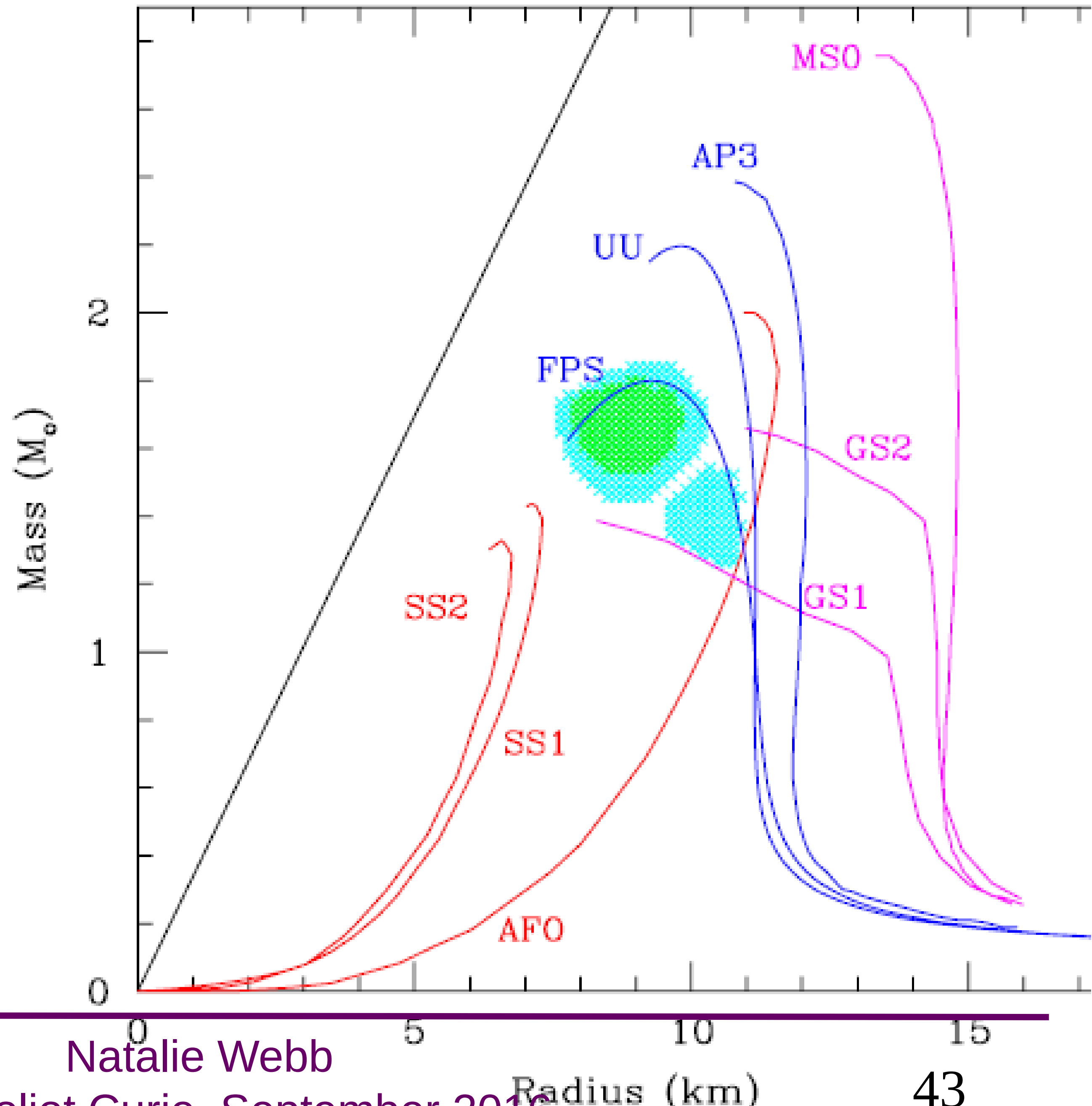
Modifications due to:
- General relativity (factor (1+z) increase)
- Nature of the material (opacity)
- Magnetic field confinement of material (opacity)

X-ray bursts due to thermonuclear explosions
Luminosity reaches Eddington limit
Photosphere expands, material cools ($<T_{X\text{-ray}}$)
Photosphere contracts, 2nd brighter peak observed

Neutron star interiors: mass & radius from bursts

Using:

$$4\pi R_*^2 F_* = L_*$$

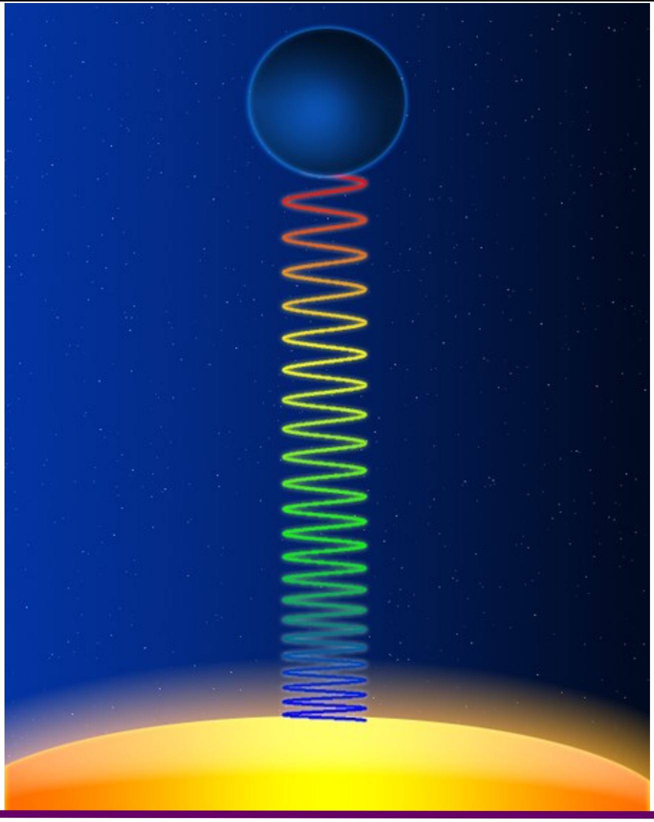


Özel et al. (2009) results from EXO 1745-248

Contours: 1 and 2 σ

Neutron star interiors : mass & radius from grav. redshift

Observable	Measurement	Dependence on NS Properties
$F_{\rm Edd}$	$(2.25 \pm 0.23) \times 10^{-8}$ erg cm^{-2} s^{-1}	$\frac{1}{4\pi D^2}\frac{4\pi GMc}{\kappa_{\rm es}}\left(1-\frac{2GM}{c^2R}\right)^{1/2}$
z	0.35	$\left(1-\frac{2GM}{Rc^2}\right)^{-1/2}-1$
$F_{\rm cool}/\sigma T_{\rm c}^4$	1.14 ± 0.10 (km/kpc)^2	$f_\infty^2\frac{R^2}{D^2}\left(1-\frac{2GM}{Rc^2}\right)^{-1}$



Neutron star interiors : mass & radius from grav. redshift

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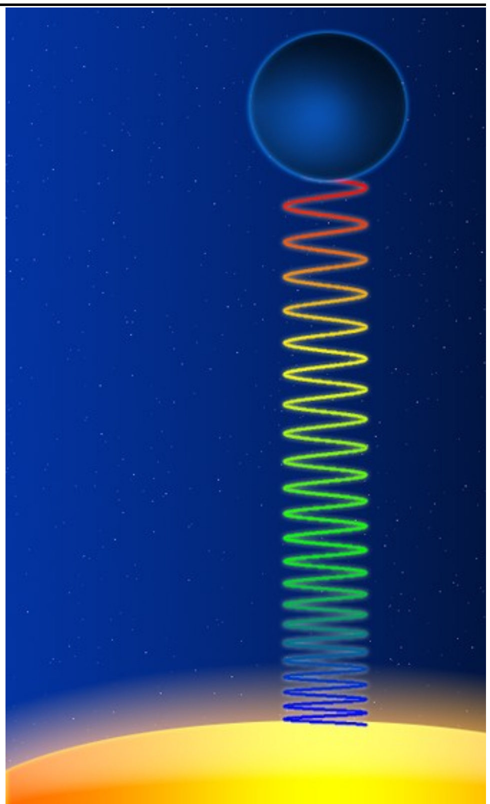

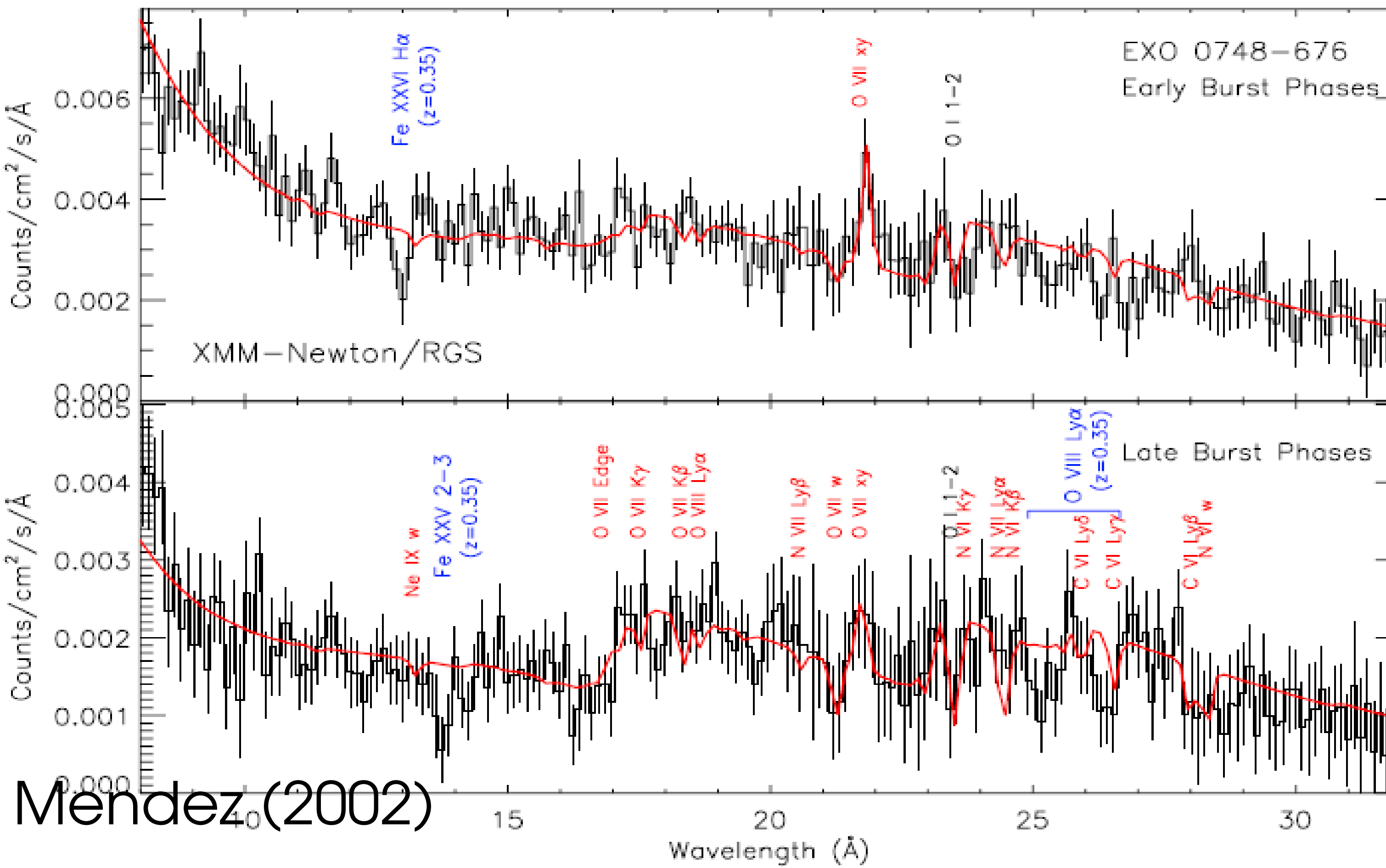

Cottam, Paerels & Mendez (2002)

Neutron star interiors : mass & radius from grav. redshift

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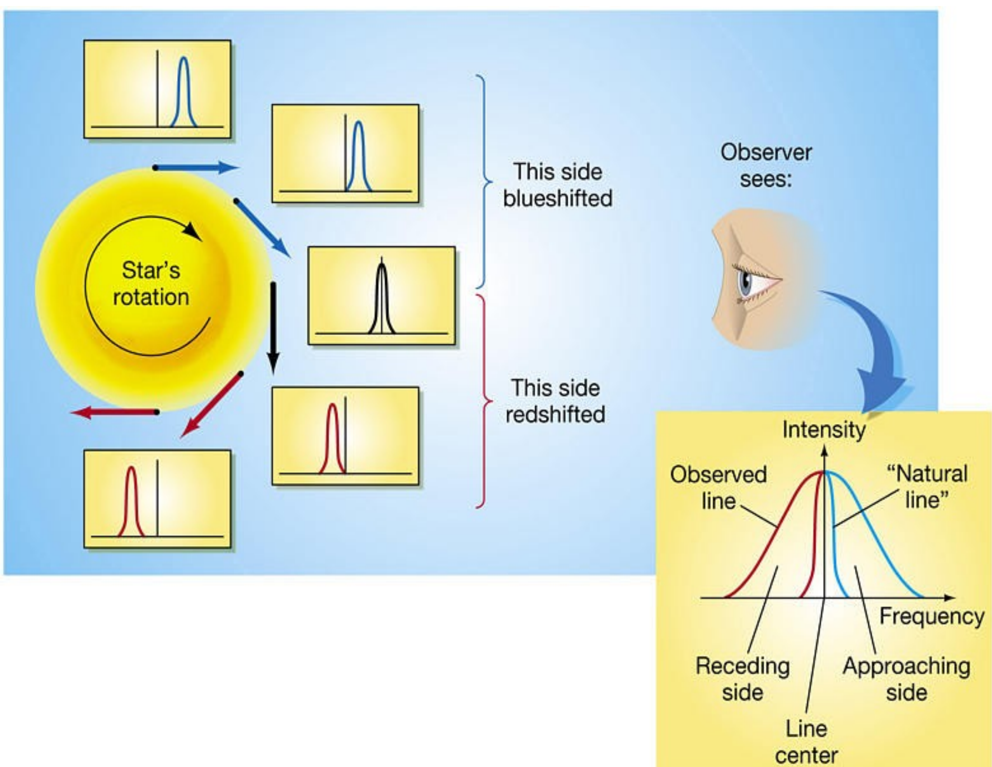

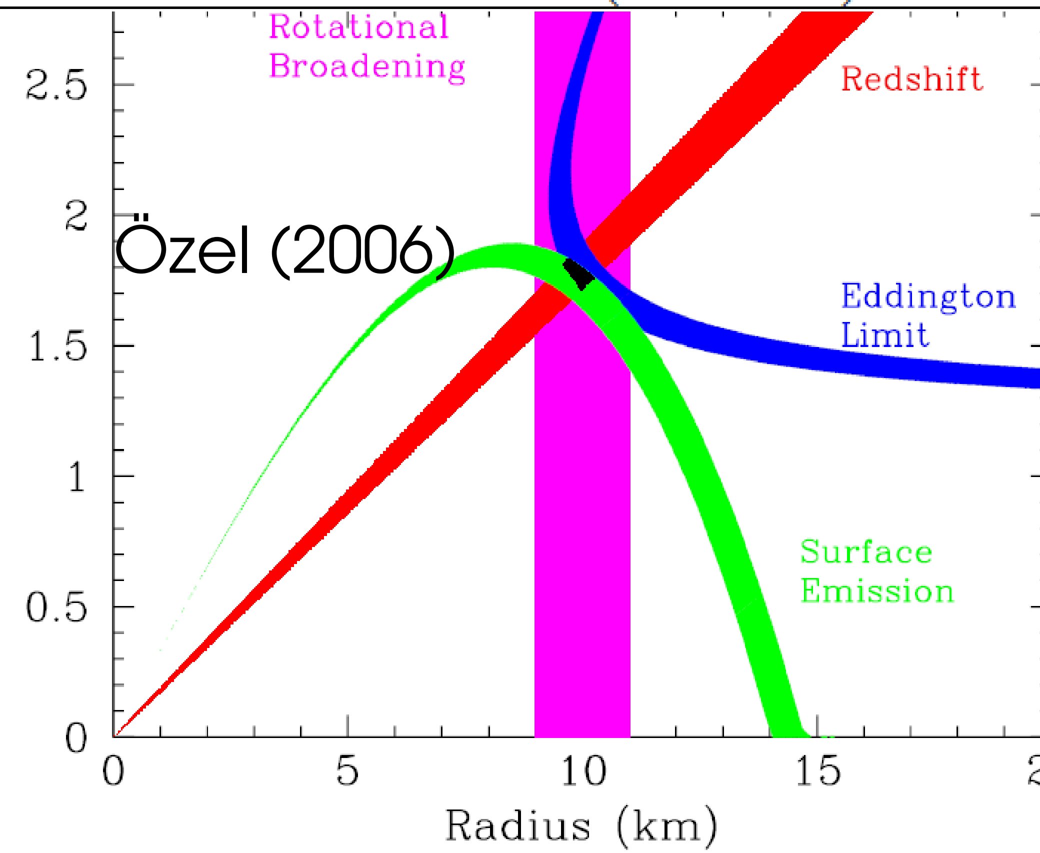

Neutron star interiors: mass & radius from thermal radiation

Globular clusters: Dense groups of old stars

Many compact objects

Binaries form due to stellar interactions

Distance to cluster well constrained

Binaries difficult to detect because of high stellar density

Neutron star interiors: mass & radius from thermal radiation

Globular clusters: Dense groups of old stars

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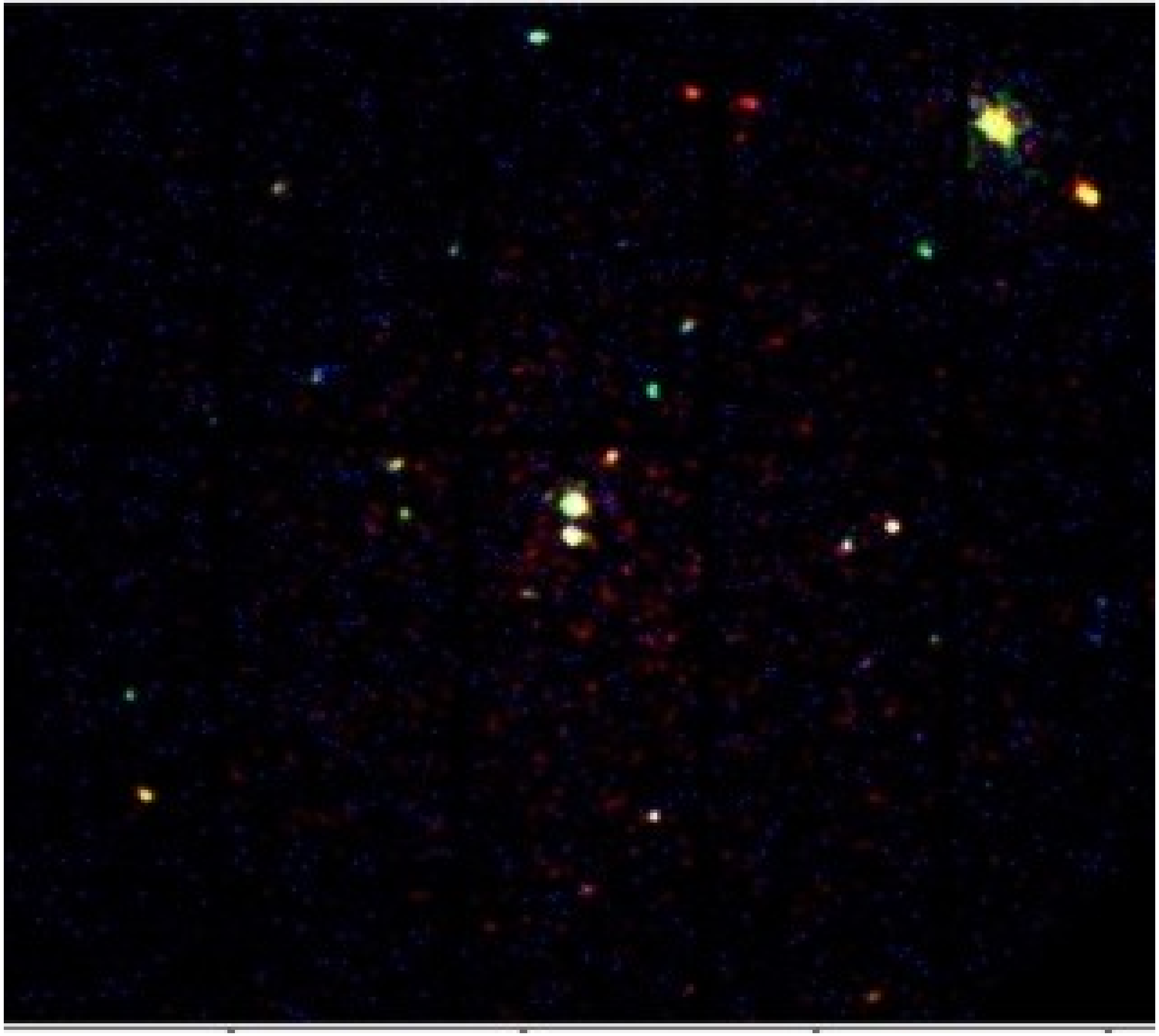


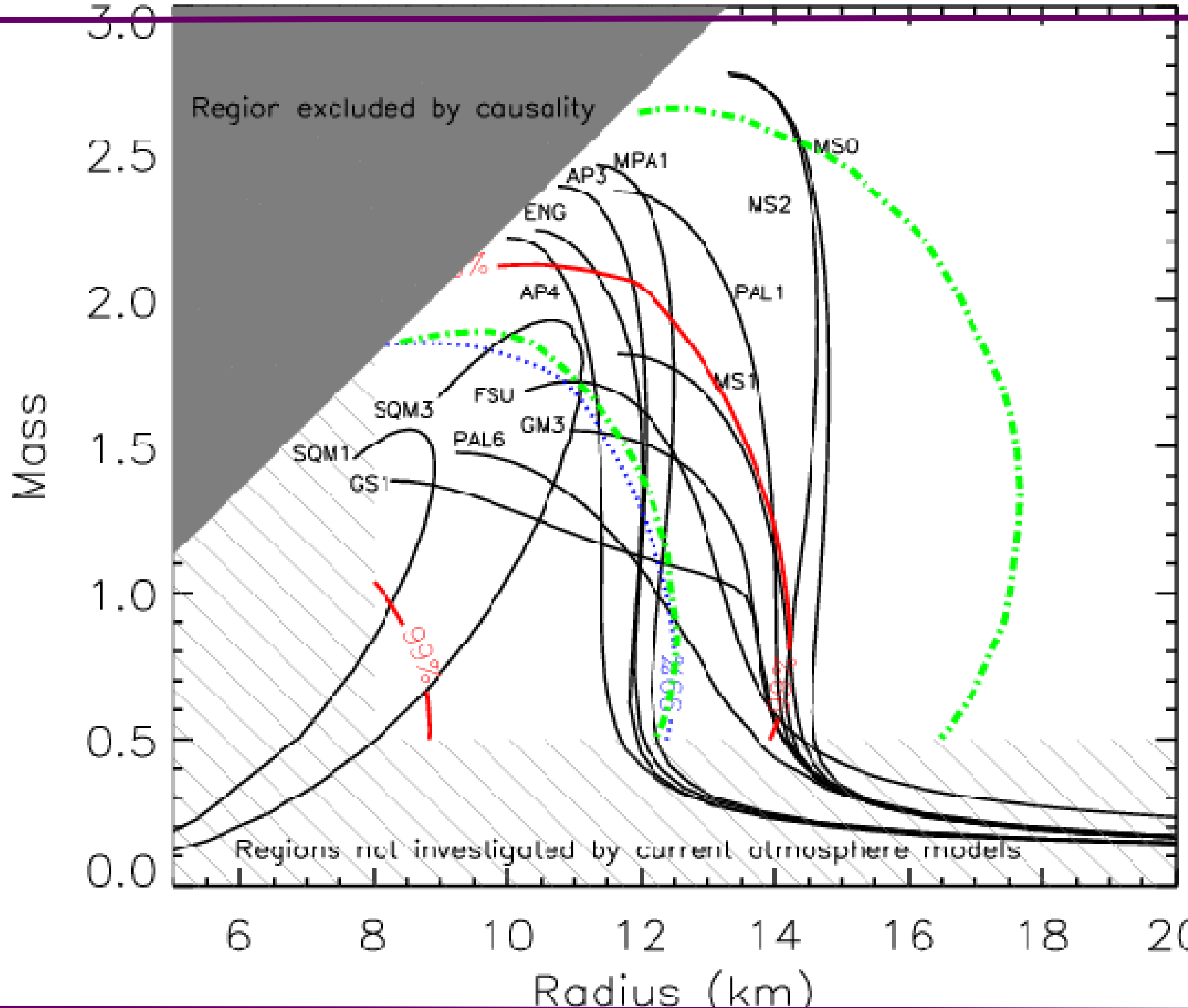
Neutron star interiors: mass & radius from thermal radiation

Mass & radius determined from X-ray spectra if distance known:

$$F_{\infty} = (R_{\infty}/d)^2 \sigma T_{\infty}^4 \quad \text{et} \quad R_{\infty} = R/(1-2GM/Rc^2)^{0.5}$$

F = flux, R = radius, d = distance, σ = Boltzman cons.
T = temperature, G = gravitational constant,
c = speed of light in vacuum

Neutron star interiors: mass & radius from thermal radiation



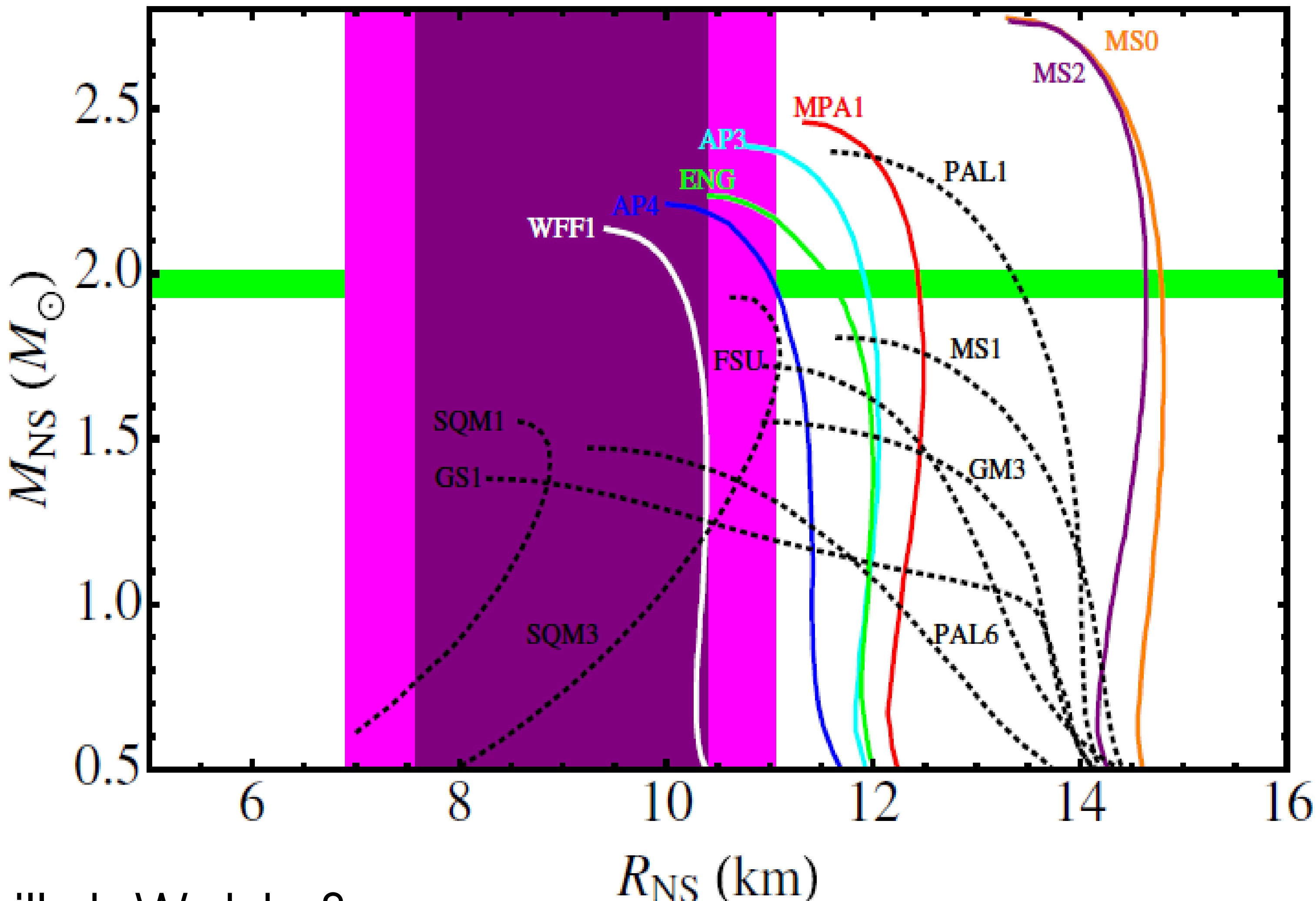
M 13

ω Cen

X7, 47 Tuc
(Heinke et al. 2006)

(Webb & Barret, 2007)

Neutron star interiors: mass & radius from thermal radiation



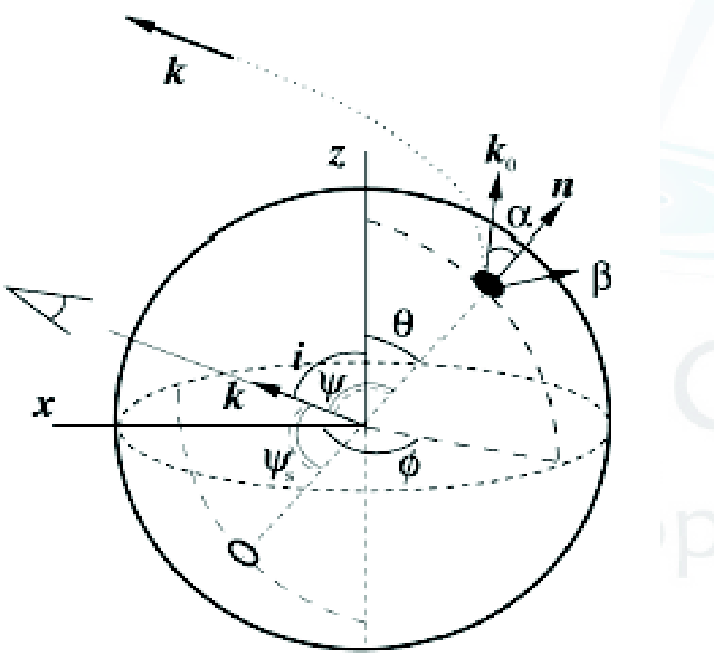
(Guillot, Servillat, Webb & Rutledge, 2013)

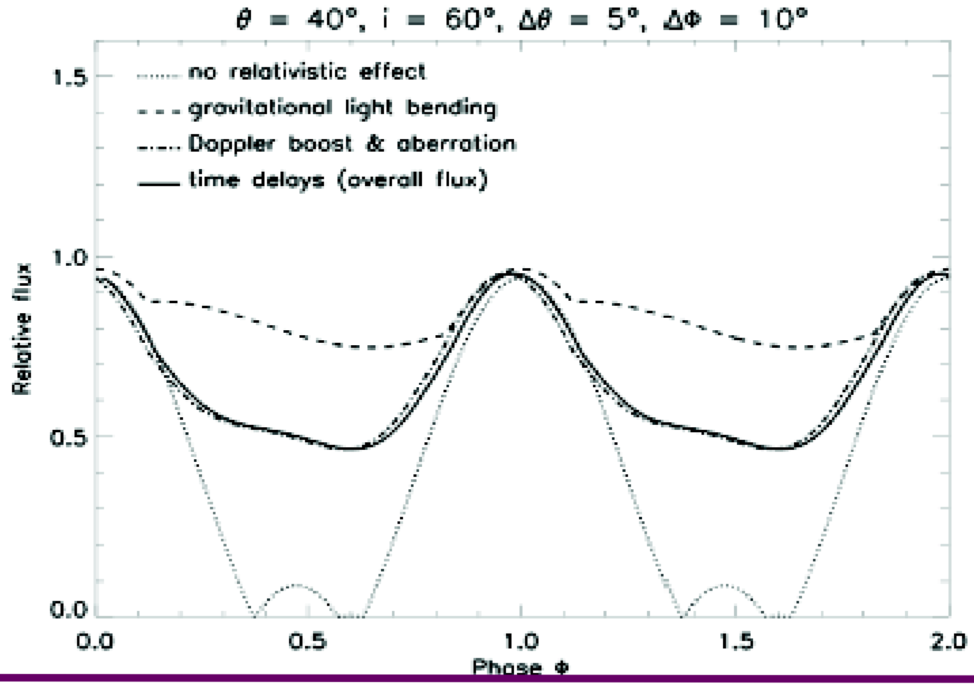
Neutron star interiors: mass & radius from thermal radiation

Modelling of thermal X-rays from MSPs

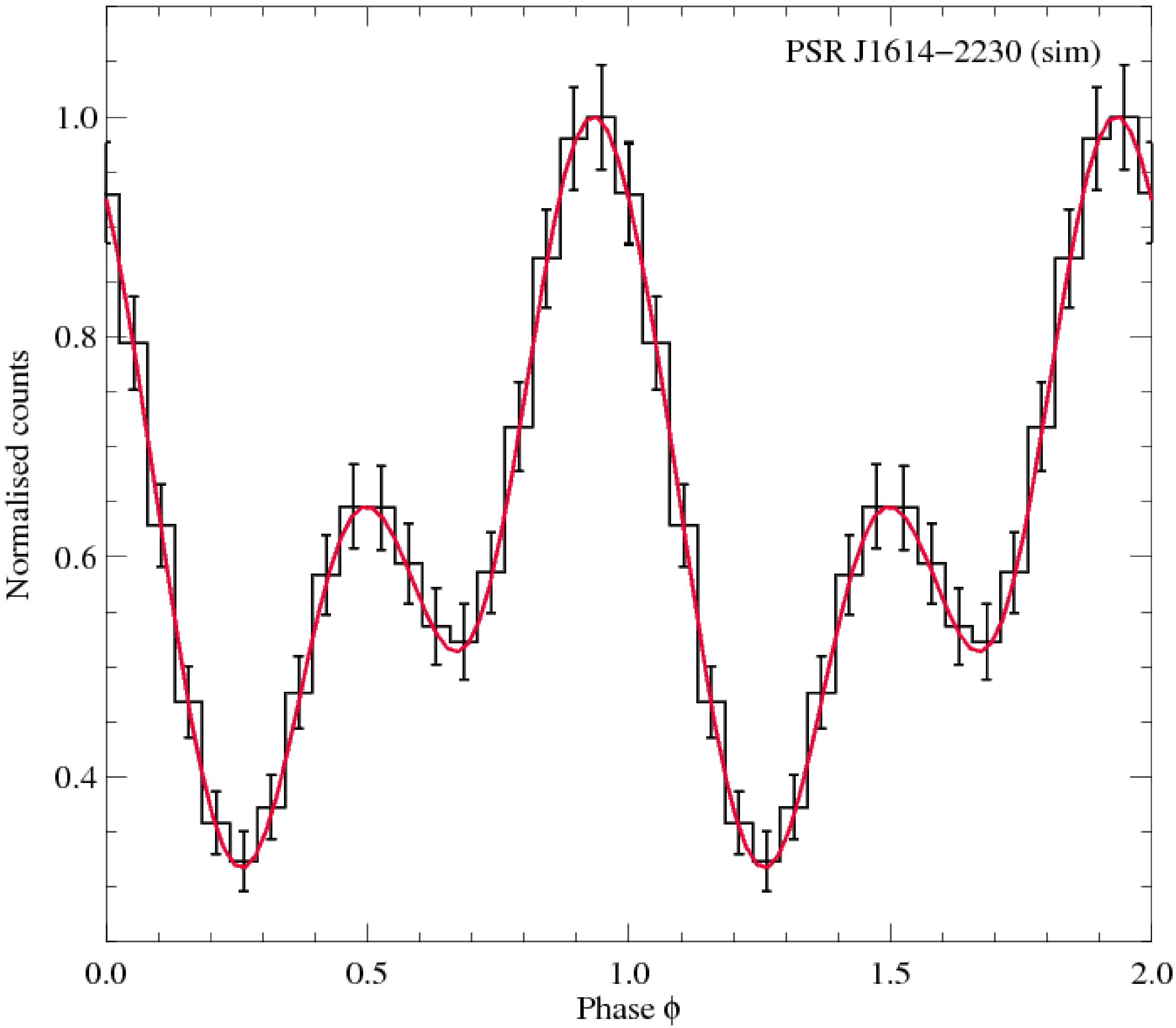
Thermal emission coming from a heated PC on a weakly magnetised NS covered by an H atmosphere (*Bogdanov et al. 2007*).

- ▶ Relativistic effects (light bending, aberration, Doppler boosting, time delays).
- ▶ Many parameters : M, R, freq, i, θ . . .
- ⇒ Reduce the number of free parameters (*Venter et al. 2009*).
- ⇒ Account for the oblateness of the NS (*Morsink et al. 2007*).
- ⇒ **Tighter constraints on M/R.**



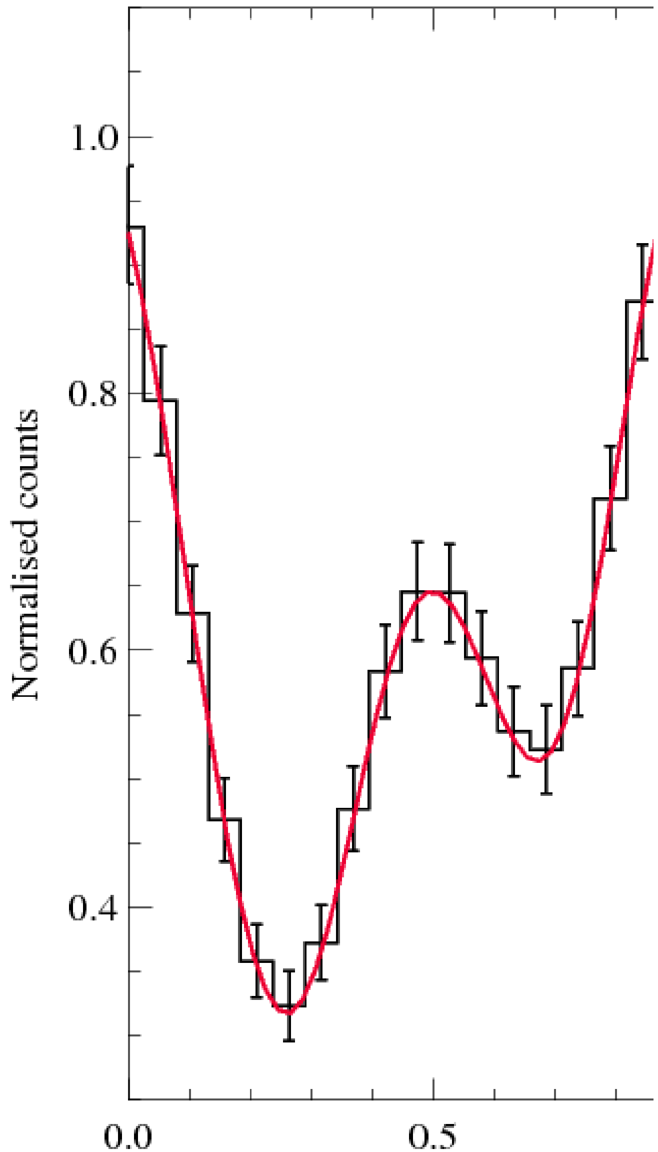


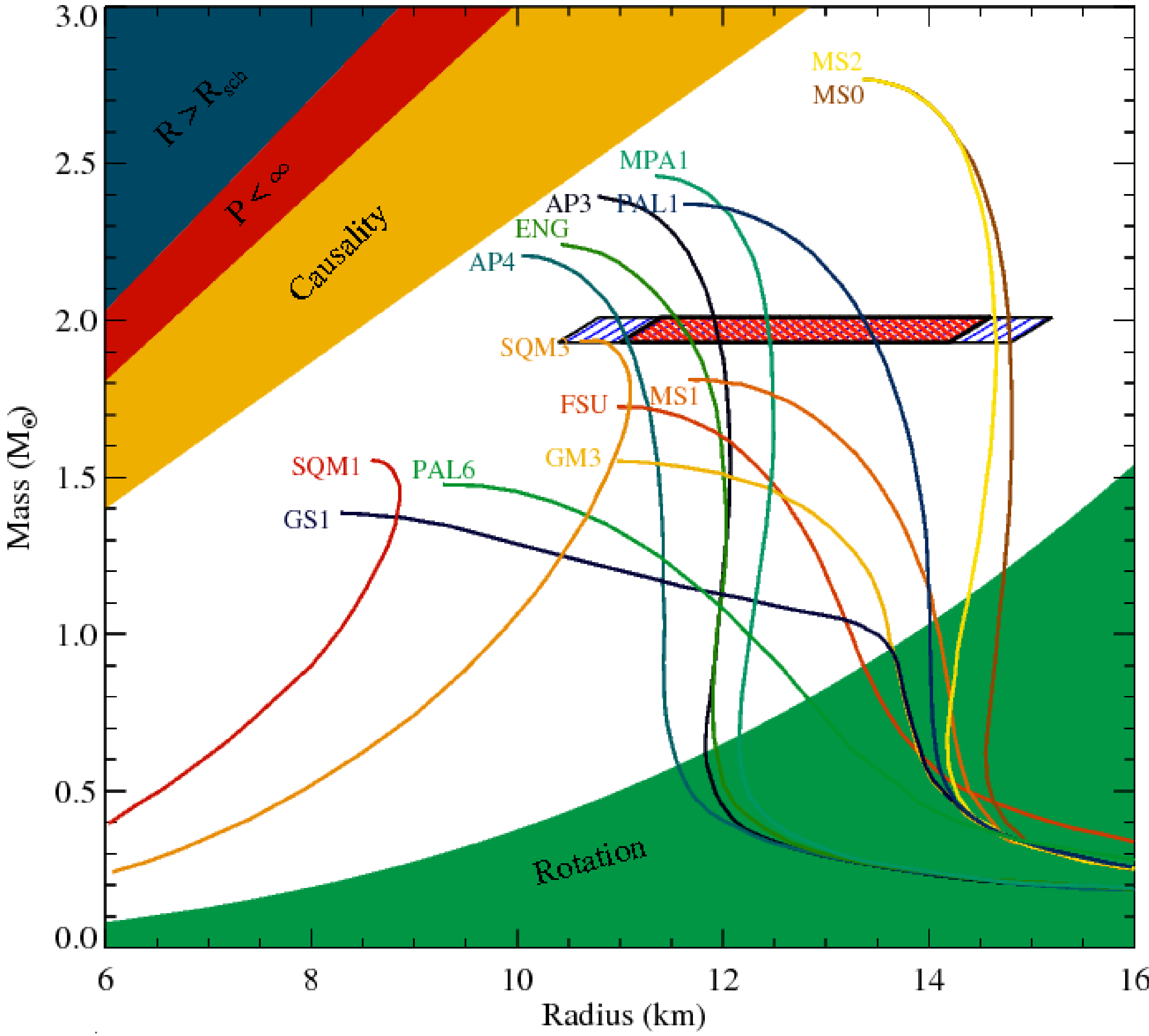
Neutron star interiors: mass & radius from thermal radiation



463 ks
PSR J1614-2230
XMM-Newton

Neutron star interiors: mass & radius from thermal radiation



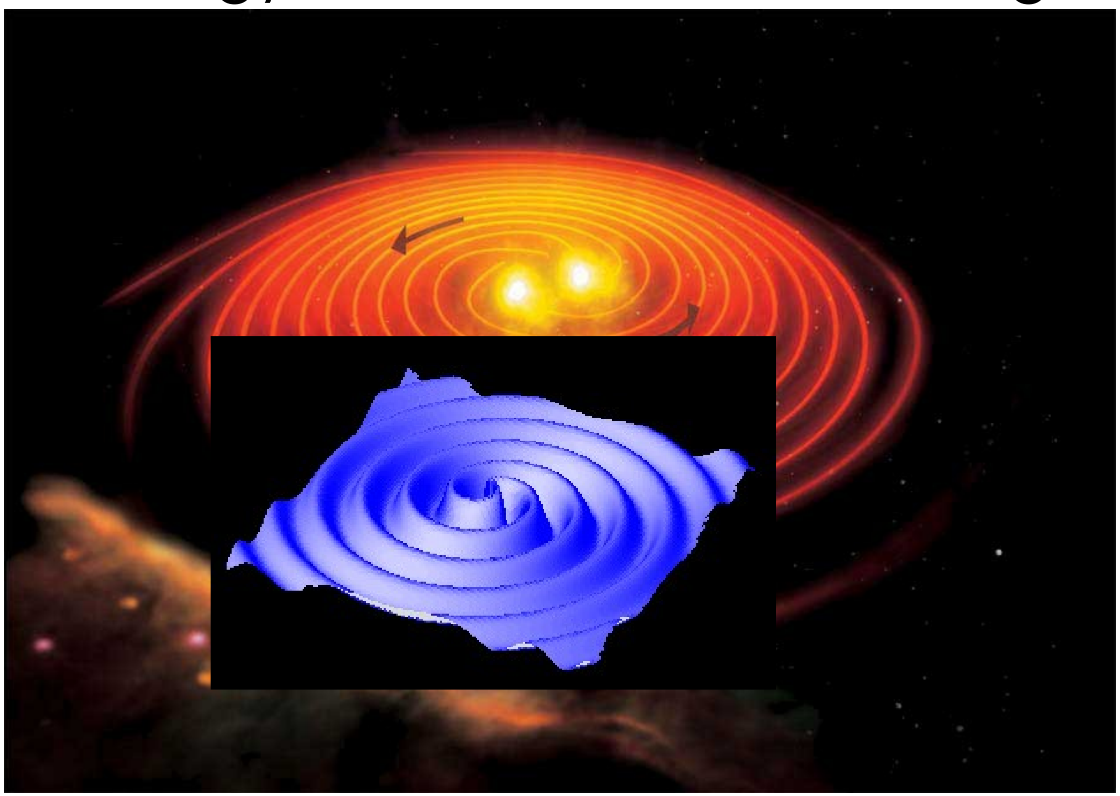


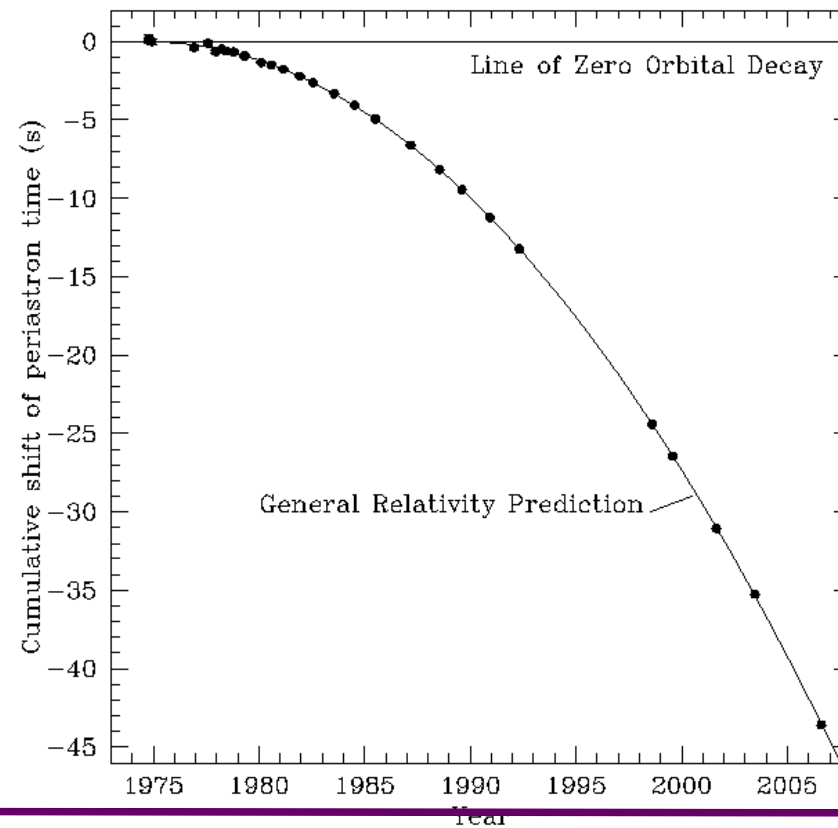
X-ray observations : Athena

Neutron star interiors: mass & radius from grav. waves

Gravitational waves : A deformation of space-time, caused by the presence of a body. They move at the speed of light.

Hulse & Taylor (1975), observed a neutron star binary (PSR 1913+16) to slow down at a rate compatible with the dissipation of energy due to the emission of gravitational waves.



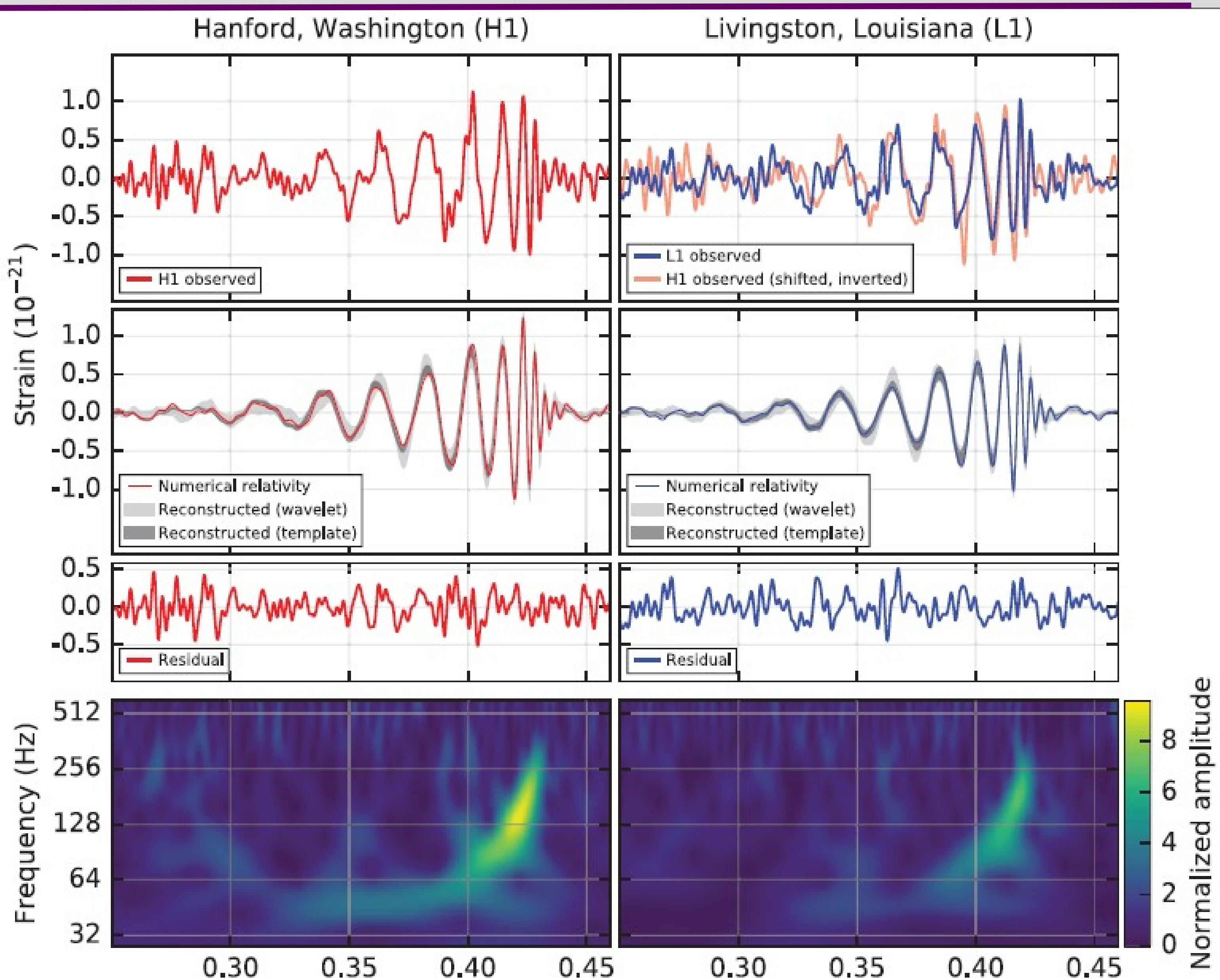


Neutron star interiors: mass & radius from grav. waves

Gravitational waves first directly detected from a black hole binary

Predict more neutron star binaries than black hole binaries

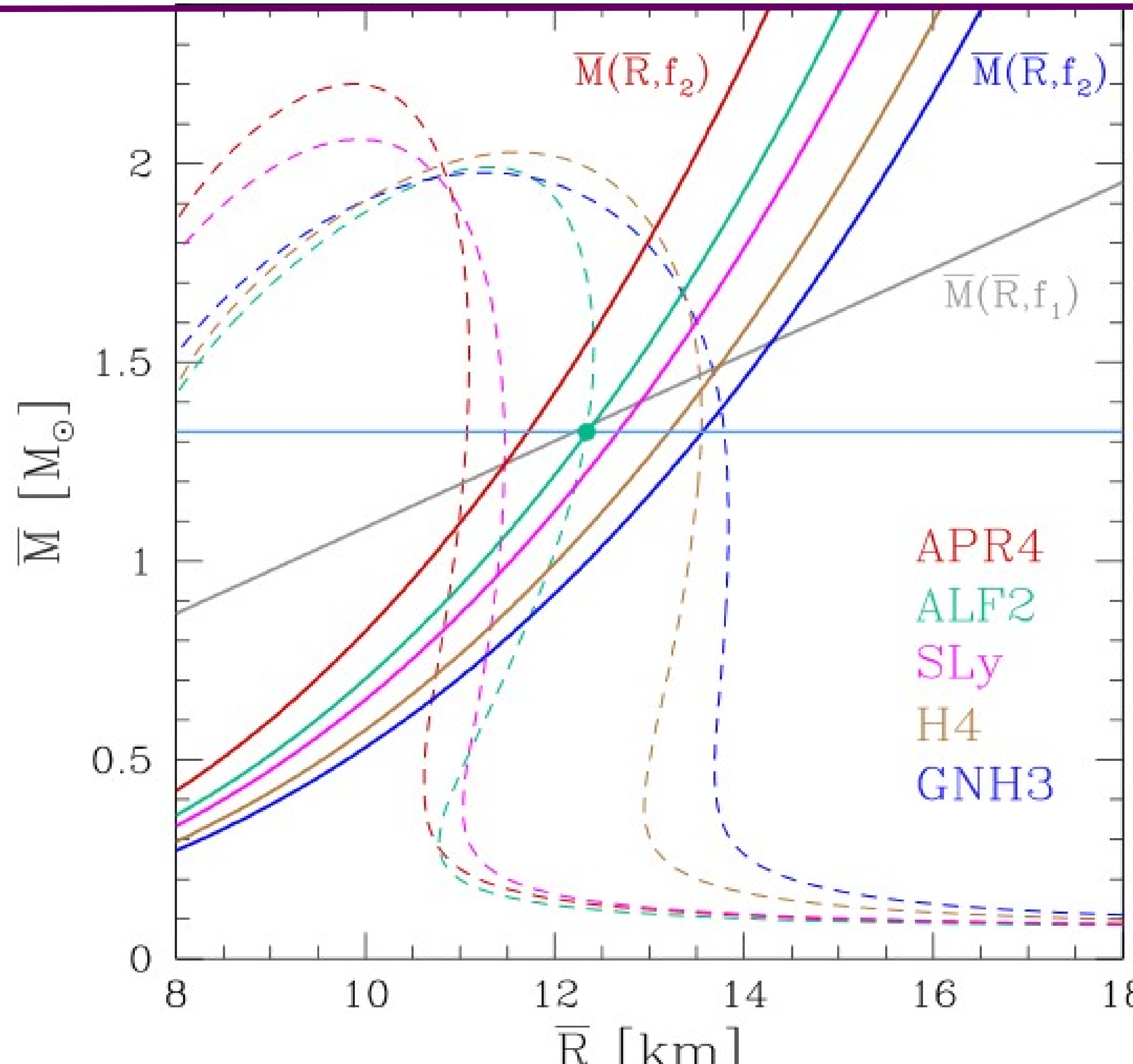
(Abbott et al. 2016)



Neutron star interiors: mass & radius from grav. waves

Solid curves show constraints possible from gravitational wave detections of double neutron star systems

(Takami, Rezzolla & Baiotti, 2014)



Summary

Multi-wavelength/messenger observations of neutron stars allow us to :

- examine phase space of nucleii not accessible elsewhere
  - cold, dense matter
  - possibly also quark-gluon plasmas

- search for exotic matter

- understand the endpoints of stellar evolution
  - constrain the massive star population
  - study highly accelerated particles

- test general relativity

Extra slides

Neutron star interiors

Neutronisation

β decay:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

At $\rho \sim 10^7$ g cm^{-3}

The reaction accelerates collapse of stellar core by consuming electrons that provided degeneracy pressure

Heavy ions loaded with neutrons are produced

At $\rho > 3\ 10^{11}$ g cm^{-3} : neutron drip (neutron ejection from atomic nucleus)

Nucleonic degeneracy in absence of other phenomena $\Rightarrow$ matter becomes soup of neutrons, protons & electrons in β equ.

$p = e^-$ and in minority w.r.t. neutrons.

Neutron star interiors

The apparition of muons

When ρ,↑the chemical potential increases, the star can become filled with μ

$$e^- \rightarrow \mu^- + \nu_e + \overline{\nu}_m$$

Neutrinos escape the star and reduce the energy, again contributing to its further collapse

The muons formed can play the same rôle as the electrons in the general Beta equlibrium.

Neutron star interiors

Hyperonisation

Strong reactions between nucleons (N) provoke the apparition of new particles, such as the hyperon lambda :

$$N + N \longrightarrow N + \mathrm{L} + K$$

If the particle does not undergo a phase change to a condensate (these are bosons), kaons can decay to form leptons & photons

The photons and neutrinos escape the star, which reduces the temperature further, meaning that the internal energy further accelerates the collapse and increases the baryon degeneracy.

Neutron star interiors

Finally, the hyperons are trapped in a « Fermi sea ».

They become stable because of the exclusion principle which forbids their decay (« Pauli blocking»).

The conservation of strangeness (zero at outset) is violated because the negative strangeness of kaons is lost (Prakash, Cooke & Lattimer, 1995).

This process continues until the whole star becomes degenerate and no further strong interaction can take place.

Neutron star interiors

The phase transition produces a plasma of free quarks

A plasma of free quarks (u d s) can exist

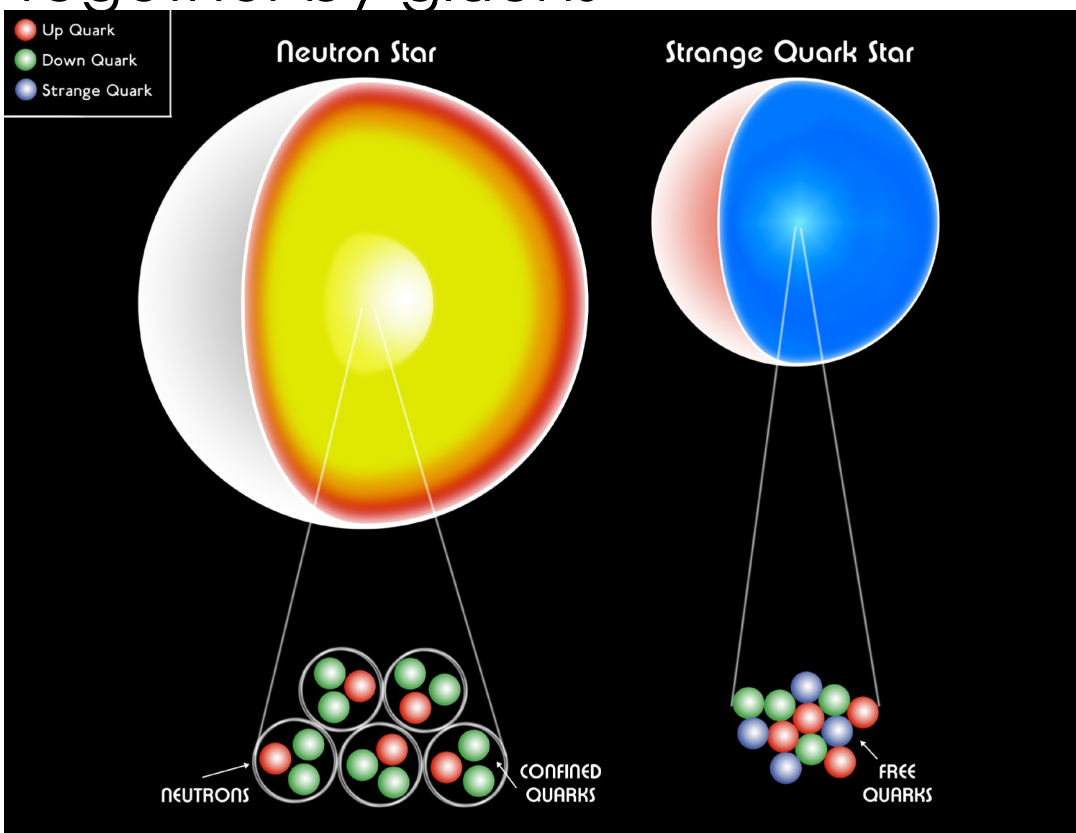
The hypothesis of strange matter stipulates that the matter is in the fundamental state (minimal energy) in extreme compactness, which gives rise to the free quarks.

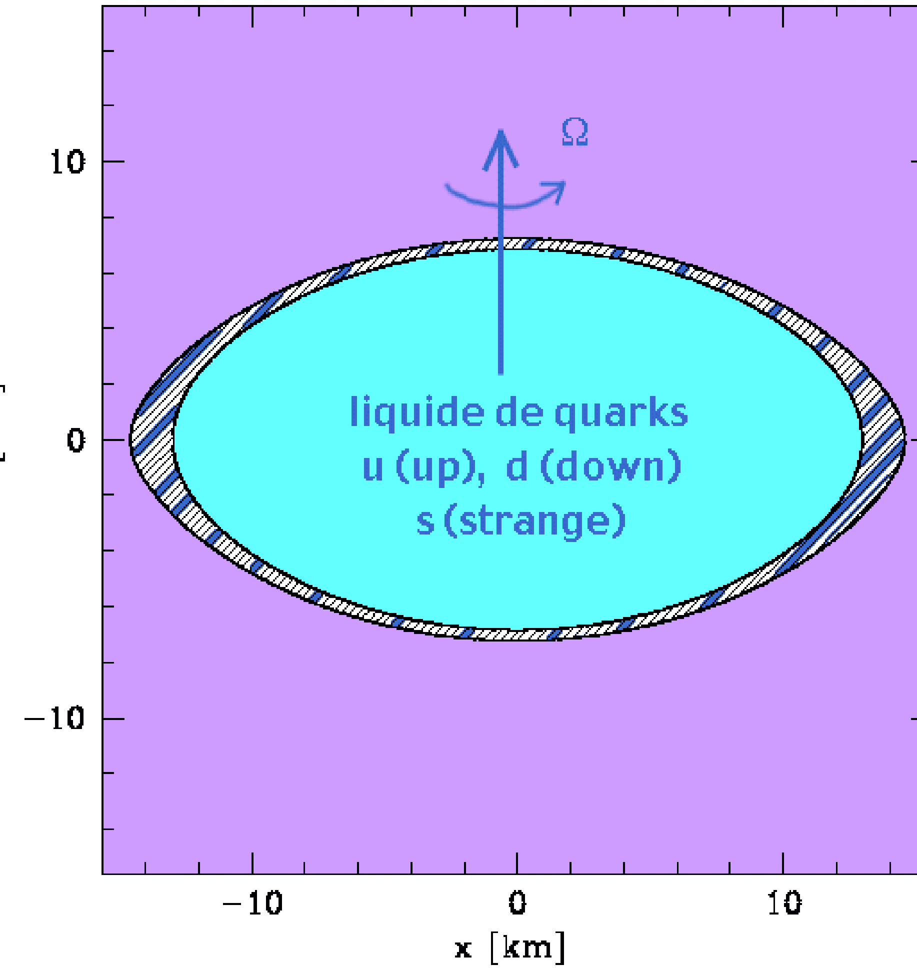
There is no formal equation of state describing the equation of state of a quark plasma and the transition from confined to free quarks is still unclear.

Neutron star interiors

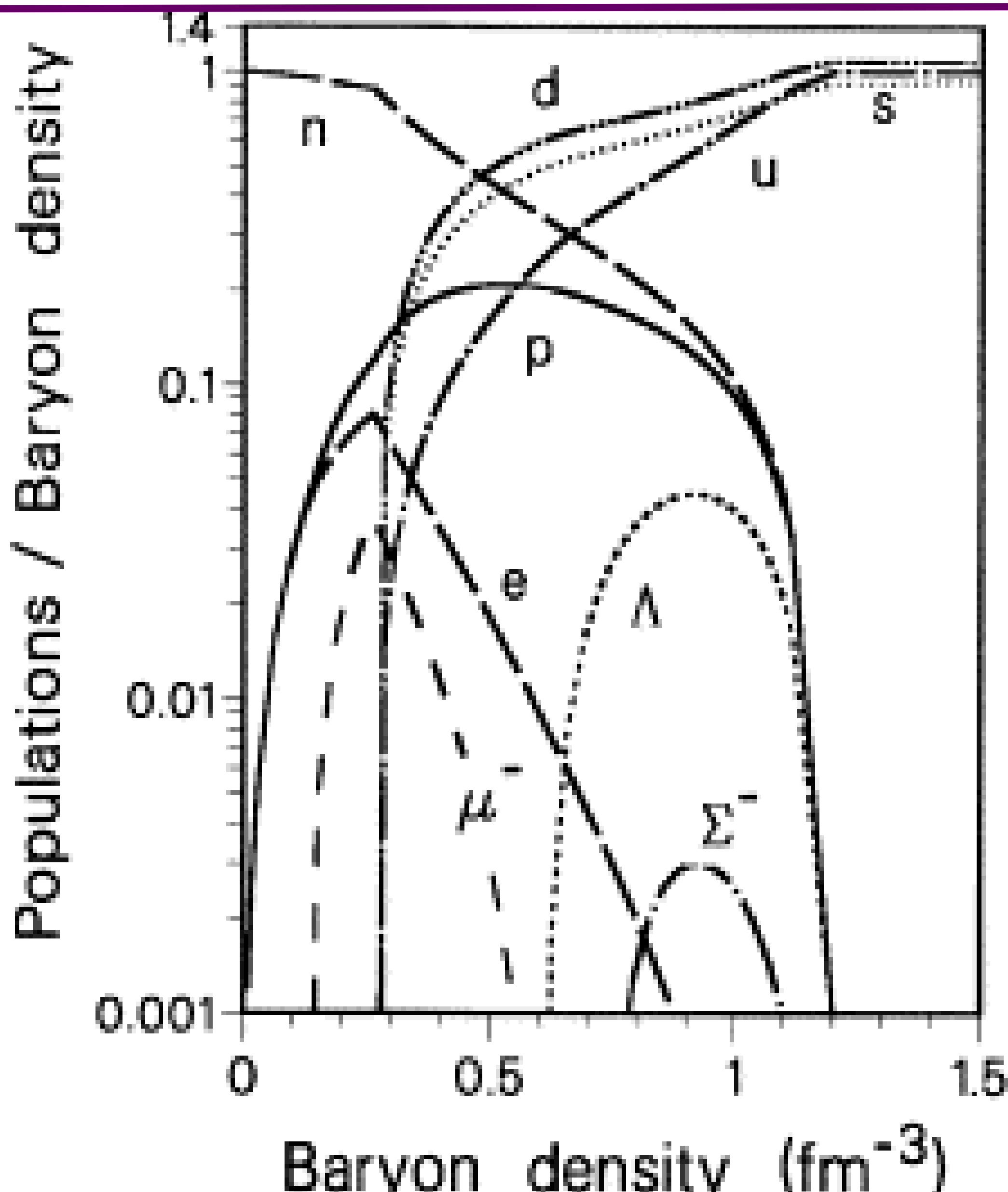
Radius of quark star
< radius of neutron star

Quark star held
together by gluons





Neutron star interiors

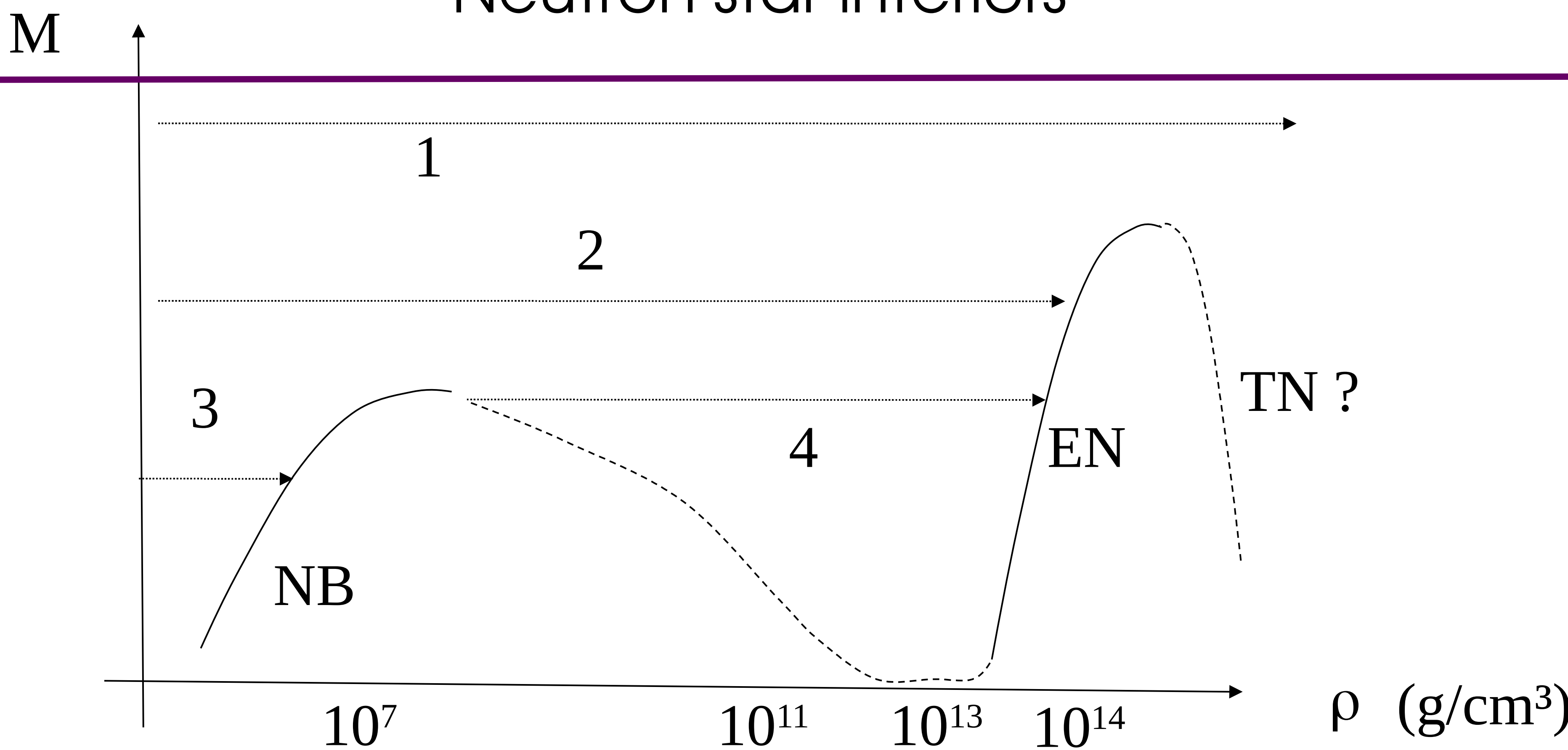


Composition of hyperon-rich hadronic matter with phase transition towards unconfined high density hadrons (quark population : u d and s).

Between 0.3 and 1.2 fm^{-3}, a mixed phase appears.

Above this density, only free quarks remain.

Neutron star interiors



1 : > 3 M_{sol}, nothing can stop collapse of space-time to a singularity (black hole) even taking into account nuclear repulsion

2 : Stellar core stabilises as neutron star.

3 : Stellar core stabilises as white dwarf

4 : White dwarf accretes to reach the Chandrasekhar mass. Formation of a neutron star.